An Introduction to Probability and Statistics
Problem Sections

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Contents

Chapter 1

Introduction

1.4 Problems

1. Give one possible sample of size 3 from each of the following populations. In other words, list 3 of the units that constitute the population described.

   (a) All the large cities in the United States;
   (b) All the cities in Louisiana;
   (c) All TV or movie streaming services;
   (d) All companies listed on the New York Stock Exchange.

2. Identify the unit on which each of the following variables is measured. For example, for the variable "the age of a person", a person is a unit since this variable is a characteristic of the person.

   (a) The number of people in a household;
   (b) The size of the screen of a laptop computer (measured in inches);
   (c) The number of credit hours a student is registered for;
   (d) The manufacturer of an automobile;
   (e) The color of the front entrance door of a house;
   (f) The classification (grade level) of a student in a high school.

   For questions 3 through 5 classify each **blue, bold faced number or description of a number** (there are two such numbers or descriptions in each problem) as a parameter or statistic (there is one of each in each problem). Carefully explain your reasoning (include a clear description of the population and the sample).

3. The receiving department of a company inspects a truckload of ball bearings to determine whether the bearings are within specified size limits. It is known, by the shipper but not by the receiving department, that the average diameter of the bearings in this truckload is **2.50**cm. The receiving department obtains a random sample of 100 ball bearings from this truckload and finds that the average diameter of these bearings is **2.55**cm.

4. A telephone sales outfit in Los Angeles uses a device that dials residential phone numbers in the city at random. Of the first 200 numbers dialed, **47%** are unlisted numbers. This is not surprising since **52%** of all Los Angeles residential phone numbers are unlisted.

5. It might be of interest to determine **the percentage of all personal vehicles registered in the United States that are more than 10 years old**. It would probably be easier to determine **the percentage of all personal vehicles registered in Louisiana that are more than 10 years old**.

6. A medical researcher wants to study the effects of a particular regimen of radiotherapy on the survival time (time until death) of patients suffering with a particular type of cancer.

   (a) What is a unit?

   (b) What is the variable of primary interest to the medical researcher?

   (c) What is the population of interest to the medical researcher.(*i.e.*, What is the target population?)

   In parts d and e you will need to describe what you believe would be a plausible sampled population and how it might differ from the target population of part c.

   (d) Describe how the researcher could select a sample from the population and describe the sampled population determined by your method of obtaining a sample.

   (e) What problems might arise in sampling from this population?

7. Ten vehicles were selected at random from a partial listing of all the vehicles that have been issued campus parking permits, and the data tabulated below were recorded. Note: The list that the sample was selected from was created a few weeks before the semester began and does not include all vehicles with campus parking permits.

Table 1.1: Characteristics of the ten vehicles

Vehicle	Type	Make	Color	Age (years)	Commute Distance (miles)
1	car	Honda	blue	8	12
2	car	Ford	white	7	30
3	truck	Chevy	brown	2	18
4	car	Toyota	white	3	6
5	truck	Toyota	black	9	4
6	motorcycle	Harley	black	3	28
7	car	Honda	green	6	45
8	truck	Ford	silver	10	10
9	motorcycle	Honda	red	1	4
10	car	Nissan	white	3	18

(a) What is a unit?

(b) What is the population of interest? (*i.e.*, What is the target population?)

(c) What is the population we have information about? (*i.e.*, What is the sampled population?)

Chapter 2

Descriptive Statistics

2.9 Problems

1. Identify the unit on which each of the following variables is measured and classify the variable as qualitative or quantitative. If the variable is qualitative, classify it as nominal or ordinal. If the variable is quantitative, classify it as inherently discrete or conceptually continuous. Be sure to explain your reasoning. For example, for the variable "the age of a person", a person is a unit and the possible values of the variable are positive numbers representing possible ages. Since these ages are numbers on the number line this is a quantitative variable. In practice, age is usually treated as a discrete quantitative variable since we usually round our age down to our most recent birth date; however, thinking of age as the amount of time since birth, we can argue that age should be viewed as a continuous variable.

   (a) The number of children in a family;
   (b) The size of the memory (RAM) in a laptop computer (measured in GB);
   (c) The number of correct answers on a multiple choice exam;
   (d) The blood type (A, AB, B, O) of a person;
   (e) The letter grade obtained by a student in a statistics class.

   Terminology note: Each of questions 2 and 3 involves two variables. One (the explanatory variable) is used to divide the units into two subgroups. The other (the response variable) is the variable of primary interest in the sense that we want to compare and contrast the distributions of the response variable for each subgroup.

2. The following table provides the number of households (in thousands) for the city of New York and the city of Los Angeles categorized by the language spoken at home for the household.

Table 2.1: Language spoken at home.

	location	
language	New York	Los Angeles
English	3982	1384
Spanish	1884	1539
Other Indo–European	998	244
Asian and Pacific Island	546	277
total	7410	3444

(a) There are 10,854 thousand units in this example. What is a unit for this example?

(b) What is the explanatory (grouping) variable in this example? What are its possible values?

(c) What is the response variable in this example? What are its possible values? Is this variable nominal qualitative, ordinal qualitative, discrete quantitative, or continuous quantitative?

3. Joe DiMaggio and Mickey Mantle were two well known baseball players. DiMaggio played center field for the New York Yankees for 13 years and was succeeded by Mantle who played center field for 18 years. There has been some argument about which of these two players was better at hitting home runs. The data given below are the numbers of home runs hit by the player during each of the seasons he played. For each player these numbers of home runs are listed in order by the seasons he played.

Table 2.2: Home run data.

Joe DiMaggio							Mickey Mantle								
29	46	32	30	31	30	21	13	23	21	27	37	52	34	42	31
25	20	39	14	32	12		40	54	30	15	35	19	23	22	18

(a) There are 31 units in this example. What is a unit for this example?

(b) What is the explanatory (grouping) variable in this example? What are its possible values?

(c) What is the response variable in this example? What are its possible values? Is this variable nominal qualitative, ordinal qualitative, discrete quantitative, or continuous quantitative?

For each of the following examples:

(a) Describe the unit and give the number of units in the entire group.

(b) Describe the variable(s) and indicate the possible values of the variable(s).

(c) Classify each variable by type (nominal qualitative, ordinal qualitative, discrete quantitative, or continuous quantitative) and if there is more than one variable identify the explanatory variable and the response variable.

4. *Example. Living arrangements of college students.* One of the questions on the 2017 National Survey of Student Engagement is: Which of the following best describes where you are living while attending college?

   (a) Residence hall, dormitory, or other campus housing (not fraternity or sorority house)
   (b) Fraternity or sorority house
   (c) Residence (house, apartment, etc.) within walking distance to the institution
   (d) Residence (house, apartment, etc.) farther than walking distance to the institution
   (e) None of the above

Table 2.3: Living arrangements of college students.

	first–year students		seniors	
response	count	percent	count	percent
On campus housing	86318	68.1	26888	15.9
Fraternity or sorority house	610	0.5	2067	1.2
Off campus – within walking	7647	6.0	40866	24.1
Off campus – beyond walking	26115	20.6	87535	51.7
None of the above	6055	4.8	12012	7.1
total	126745	100	169368	100

Note: Through its student survey, The College Student Report, the National Survey of Student Engagement (NSSE) annually collects information at hundreds of four-year colleges and universities about first-year and senior students' participation in programs and activities that institutions provide for their learning and personal development. The results provide an estimate of how undergraduates spend their time and what they gain from attending college. In 2017, 725 colleges and universities participated and 517,850 students completed the survey.

5. *Example. Opinions regarding United States scientific achievements.* The data which follow come from a Pew Research Center general public science survey conducted August 15–25, 2014 and an AAAS survey of scientists conducted September 11 – October 13, 2014. The question considered here is:

   Do you think the U.S. is the best in the world, above average, average, or below average in its scientific achievements compared to other industrialized countries?

   Table 2.4: Opinions regarding United States scientific achievements.

	US Adults		AAAS Scientists	
response	count	percent	count	percent
Best in the world	344	18	1687	45
Above average	783	40	1762	47
Average	668	34	225	6
Below average	156	8	74	2
total	1951	100	3748	100

   Note: The Pew Research Center is a nonpartisan fact tank that informs the public about the issues, attitudes and trends shaping the world. It conducts public opinion polling, demographic research, media content analysis and other empirical social science research. Pew Research Center does not take policy positions. It is a subsidiary of The Pew Charitable Trusts. The American Association for the Advancement of Science (AAAS) is the world's largest multidisciplinary scientific professional society. Established in 1848, the AAAS publishes Science magazine, one of the most widely circulated peer-reviewed scientific journals in the world. It is an open-member organization that brings together a wide segment of the scientific community.

6. *Example. Vole reproduction.* An investigation was conducted to study reproduction in laboratory colonies of voles. This example is taken from Devore and Peck, *Statistics*, (1997), 33; the original reference is the article "Reproduction in laboratory colonies of voles", *Oikos*, (1983), 184. The data summarized in the table below are the numbers of babies in 170 litters born to voles in a particular laboratory.

Table 2.5: Vole baby frequency distribution.

number of babies	frequency
1	1
2	2
3	13
4	19
5	35
6	38
7	33
8	18
9	8
10	2
11	1
total	170

7. *Example. Homophone confusion and Alzheimer's disease.* A study was conducted to investigate the relationship between Alzheimer's disease and homophone spelling confusion. A homophone pair is a pair of words with the same pronunciation having different meanings and spellings. Twenty patients with Alzheimer's disease were asked to spell 24 homophone pairs (given in random order) and the number of homophone confusions, *e.g.* spelling doe given the context bake bread dough, was recorded for each patient. One year later, the same patients were again asked to spell the same 24 homophone pairs and the number of homophone confusions was again recorded. The data given below are the numbers of homophone confusions at the two times of measurement for the 20 Alzheimer's patients. This example comes from Neils, J., D.P. Roeltgen, and F. Constantinidou, "Decline in homophone spelling associated with loss of semantic influence on spelling in Alzheimer's disease", *Brain and Language*, **49**, (1995).

Table 2.6: Alzheimer's homophone confusion data.

patient	time1	time2	patient	time1	time2
1	5	5	11	7	10
2	1	3	12	0	3
3	0	0	13	3	9
4	1	1	14	5	8
5	0	1	15	7	12
6	2	1	16	10	16
7	5	6	17	5	5
8	1	2	18	6	3
9	0	9	19	9	6
10	5	8	20	11	8

8. *Example. Wooly–bear caterpillar cocoons.* A study was conducted to investigate the relationship between air temperature and the temperature inside a wooly–bear caterpillar cocoon. It seems quite reasonable to expect the temperature inside a cocoon to be higher than the air temperature (outside the cocoon). The data given below are pairs of air and cocoon temperatures made on 12 days at a location in the high arctic region. Each cocoon temperature is actually the average of two cocoon temperatures. This example comes from Kevan, P.C., T.S. Jensen, and J.D. Shorthouse, "Body temperatures and behaviorial thermoregulation of high arctic wooly–bear caterpillars and pupae (*Gynaephora rossii,* Lymantridae: Lepidoptera) and the importance of sunshine", *Arctic and Alpine Research*, **14**, (1982).

Table 2.7: Wooly–bear temperature data.

day	cocoon temp	air temp	day	cocoon temp	air temp
1	15.1	10.4	7	3.6	1.7
2	14.6	9.2	8	5.3	2.0
3	6.8	2.2	9	7.0	3.0
4	6.8	2.6	10	7.1	3.5
5	8.0	4.1	11	9.6	4.5
6	8.7	3.7	12	9.5	4.4

9. *Example. Birth rates for the United States in 2014.* The *National Vital Statistics Report* (Vol. 64, No. 12, December 23, 2015) presents 2014 data on U.S. births according to a wide variety of characteristics. The data used in this example are based on 100% of the birth certificates filed in all states and the District of Columbia (DC). These data are provided to the Centers for Disease Control and Prevention's National Center for Health Statistics (NCHS) through the Vital Statistics Cooperative Program (VSCP).

Table 2.8: Birthrates 2014 (births per 1,000 women).

state	birthrate	state	birthrate
Alabama	12.3	Montana	12.1
Alaska	15.5	Nebraska	14.2
Arizona	12.9	Nevada	12.6
Arkansas	13.0	New Hampshire	9.3
California	13.0	New Jersey	11.6
Colorado	12.3	New Mexico	12.5
Connecticut	10.1	New York	12.1
Delaware	11.7	North Carolina	12.2
District of Columbia	14.4	North Dakota	15.4
Florida	11.1	Ohio	12.0
Georgia	13.0	Oklahoma	13.8
Hawaii	13.1	Oregon	11.5
Idaho	14.0	Pennsylvania	11.1
Illinois	12.3	Rhode Island	10.3
Indiana	12.7	South Carolina	11.9
Iowa	12.8	South Dakota	14.4
Kansas	13.5	Tennessee	12.5
Kentucky	12.7	Texas	14.8
Louisiana	13.9	Utah	17.4
Maine	9.5	Vermont	9.8
Maryland	12.4	Virginia	12.4
Massachusetts	10.7	Washington	12.5
Michigan	11.5	West Virginia	11.0
Minnesota	12.8	Wisconsin	11.7
Mississippi	12.9	Wyoming	13.2
Missouri	12.4		

Chapter 3

Probability

3.5 Problems

1. Two family vehicles, a car and a truck, are classified by region of assembly. The three assembly regions are North America (N), Europe (E) and Asia (A). We will refer to vehicles assembled in North America as domestic and those assembled in Europe or Asia as foreign.

   (a) List the 9 elementary outcomes for this experiment (Explain your notation).
   (b) List the outcomes in the event that one of the vehicles is domestic and the other is foreign.
   (c) List the outcomes in the event that at least one of the vehicles is foreign.
   (d) List the outcomes in the complement of the event in (c).

2. Each of four restaurants is classified as local, L, or in a national/regional chain, N.

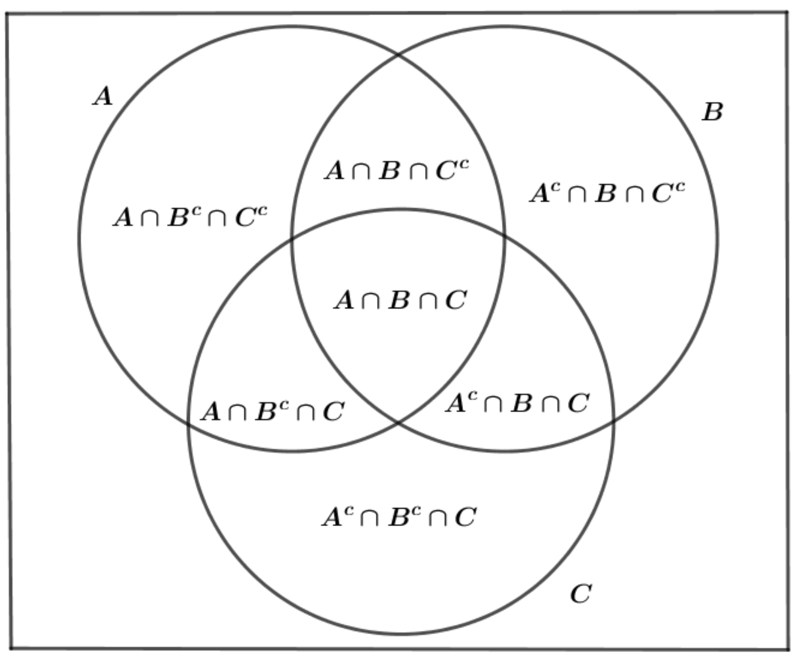
   (a) List are the 16 elementary outcomes for this experiment (Explain your notation.).
   (b) List the outcomes in the event that exactly three of the restaurants are local.
   (c) List the outcomes in the event that all four restaurants are of the same type.
   (d) List the outcomes in the event that at most one of the four restaurants are local.
   (e) List the outcomes in the union of the events in (c) and (d), and list the outcomes in the intersection of these two events.
   (f) List the outcomes in the union of the events in (b) and (c), and list the outcomes in the intersection of these two events.
   (g) List the outcomes in the complement of the event in (d)?

3. A large construction company is currently working on three large projects (1,2,3). For $i = 1, 2, 3$, let A_i denote the event that project i is completed by its contract date. For each of the events described below, use the operations of unions, intersections, and complements to describe the event in terms of A_1, A_2, and A_3, draw a Venn diagram with three circles (similar to that shown below), and shade the sections corresponding to the event.

   (a) At least one project is completed by its contract date.
   (b) All three projects are completed by their contract dates.
   (c) Only project 1 is completed by its contract date.
   (d) Exactly one project is completed by its contract date.
   (e) Project 1 is completed by its contract date or both of the other projects (but not all three) are completed by its contract date.

Figure 3.1: Venn Diagram showing the partition of the union of events A, B, and C

$$\begin{aligned} \boldsymbol{A \cup B \cup C} &= \boldsymbol{(A \cap B^c \cap C^c) \cup (A \cap B \cap C^c) \cup (A^c \cap B \cap C^c)} \\ &\quad \boldsymbol{\cup (A \cap B \cap C) \cup (A \cap B^c \cap C) \cup (A^c \cap B \cap C) \cup (A^c \cap B^c \cap C)} \end{aligned}$$



4. Suppose that a die is tossed twice. Express the following events as subsets of the set of 36 ordered pairs (see Table 3.1 reproduced below) in the sample space for this experiment. If possible describe the location of the ordered pairs instead of listing them, *e.g.*, the outcomes in column two correspond to observing a two on the second toss.

   (a) A "the number on the first toss is larger than the number on the second toss"
   (b) B "the number on the first toss is even"
   (c) C "the first toss yields a four"
   (d) D "the sum of the numbers on the two tosses is seven"
   (e) $A \cup C$
   (f) $A \cap D$
   (g) C^c

Table 3.1: Elementary outcomes (ordered pairs) when a die is tossed twice

	second toss					
first toss	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

5. A vehicle arriving at an intersection can turn left, turn right, or go forward through the intersection. For the following questions let L denote "turns left", R denote "turns right", and F denote "goes forward through the intersection." Consider an experiment consisting of observing the movement of two vehicles through the intersection.

   (a) List the elementary outcomes in the sample space for this experiment.
   (b) List the outcomes in the event that both vehicles turn.
   (c) List the outcomes in the event that the vehicles turn in opposite directions.
   (d) List the outcomes in the event that the first vehicle turns left.
   (e) List the outcomes in the union of the events in (c) and (d), and list the outcomes in the intersection of these two events?

6. Suppose that one card is selected from a deck of 20 cards that contains 10 red cards numbered from 1 to 10 and 10 blue cards numbered from 1 to 10.

   (a) List 20 elementary outcomes in the sample space Ω of this experiment.
   (b) List the outcomes in the event A "a card with an even number is selected."
   (c) List the outcomes in the event B "a blue card is selected."
   (d) List the outcomes in the event C "a card with a number less than five is selected"

   Describe each of the following events both in words and as a subset of Ω:

   (e) $A \cap B \cap C$
   (f) $A \cup B \cup C$
   (g) $A \cap (B \cup C)$
   (h) $B \cap C^c$
   (i) $A^c \cap B^c \cap C^c$.

Chapter 4

Probability Measures

4.4 Problems

1. At a certain company, 60% of the employees are certified to operate machine A, 30% are certified to operate machine B, and 10% are certified to operate both machine A and machine B. Let A denote the event that a randomly chosen employee is certified to operate machine A, and let B denote the event that a randomly chosen employee is certified to operate machine B.

    (a) Find the probability that a randomly chosen employee is certified to operate at least one of machines A and B.
    (b) Find the probability that a randomly chosen employee is certified to operate one of machines A and B but not both.
    (c) Find the probability that a randomly chosen employee is certified to operate machine A but not machines B.
    (d) Find the probability that a randomly chosen employee is certified to operate at most one of machines A and B.

2. The water supplies of the cities in a state were tested for two kinds of impurities commonly found in water. It was found that 20% of the water supplies had neither sort of impurity, 40% had an impurity of type A, and 50% had an impurity of type B. If a city is chosen at random, what is the probability its water supply has exactly one type of impurity?
3. John is going to graduate from a university at the end of the semester. After being interviewed at two companies he likes, he assesses that his probability of getting an offer from company A is 0.8, and his probability of getting an offer from company B is 0.6. If he believes that the probability that he will get offers from both companies is 0.5, what is the probability that he will get an offer from at least one of these two companies?

4. An automobile manufacturer is required to recall all its cars manufactured in a given year for the repair of possible defects in the air–bag system and possible defects in the brake system. Dealers have been notified that 3% of the cars have defective air-bag systems only, and that 6% of the cars have defective brake systems only. If 87% of the cars have neither defect, what percentage of the cars have both defects?

5. From past experience, a stockbroker believes that under present economic conditions a customer will invest in tax-free bonds with a probability of 0.6, will invest in mutual funds with a probability of 0.3, and will invest in both tax–free bonds and mutual funds with a probability of 0.15. At this time, find the probability that a customer will invest

   (a) in tax-free bonds but not mutual funds;
   (b) in tax-free bonds or mutual funds (or both);
   (c) in neither tax-free bonds nor mutual funds.

6. At a certain company, 40% of the employees are certified to operate machine A, 50% are certified to operate machine B, 40% are certified to operate machine C, 15% are certified to operate machines A and B, 10% are certified to operate machines A and C, 5% are certified to operate all three machines, and 15% are certified to operate machine C but neither machine A nor machine B. Let A denote the event that a randomly chosen employee is certified to operate machine A, let B denote the event that a randomly chosen employee is certified to operate machine B, and let C denote the event that a randomly chosen employee is certified to operate machine C.

   For each of the following, include an expression for the event in terms of set operations on A, B and C, and justify your answer.

   (a) Find the probability that a randomly chosen employee is certified to operate at exactly one of the three machines.
   (b) Find the probability that a randomly chosen employee is certified to operate machine A and machine C but not machines B.
   (c) Find the probability that a randomly chosen employee is certified to operate two of the three machines but not all three.
   (d) Find the probability that a randomly chosen employee is certified to operate at least one of the three machines.

7. Among the students in a high school graduating class, 54% studied mathematics during their senior year, 69% studied history during their senior year, and 35% studied both mathematics and history during their senior year. If one of these students is selected at random, find the probability that during their senior year

   (a) the student studied mathematics or history (or both);
   (b) the student studied neither of these subjects;
   (c) the student studied history but not mathematics.

8. Given events A and B, defined for the same sample space, is it possible to have $P(A) = 1/2$, $P(A \cap B) = 1/3$ and $P(B) = 1/4$?

9. Explain why $P(A \cup B) \leq P(A) + P(B)$.

10. Given events A and B, defined for the same sample space, with $P(A) = P(B)$ and $P(A^c \cap B^c) = P(A \cap B) = 1/6$. Find

    (a) $P(A)$.

    (b) $P(A^c \cup B^c)$.

    (c) the probability that exactly one of the events A or B occurs.

Chapter 5

Combinatorics

5.8 Problems

1. The lineup or batting order for a baseball team is a list of the nine players on the team indicating the order in which they will bat during the game.
    (a) How many lineups are possible?
    (b) If lineup is restricted so that the catcher is listed first and the pitcher is listed last, how many lineups are possible?
2. In how many ways can one plumber, one electrician, and one carpenter be selected when there are 5 choices of plumber, 3 choices of electrician, and 7 choices of carpenter?
3. (Akritas) The clock rate of a CPU (central processing unit) chip refers to the frequency, measured in megahertz (MHz), at which it functions reliably. CPU manufacturers typically categorize (bin) CPUs according to their clock rate and charge more for CPUs that operate at higher clock rates. A chip manufacturing facility will test and bin each of the next 10 CPUs in four clock rate categories denoted by G1, G2, G3, and G4.
    (a) How many possible outcomes of this binning process are there?
    (b) How many of the outcomes have three CPUs classified as G1, two classified as G2, two classified as G3, and three classified as G4?
    (c) If the outcomes of the binning process are equally likely, what is the probability of the event described in part (b)?

4. A president, treasurer, and secretary, all different, are to be chosen from a club consisting of 10 people, say persons $A, B, C, \ldots, J$. How many different choices of officers are possible if
   (a) There are no restrictions?
   (b) A and B will not serve together?
   (c) C and D will serve together or not at all?
   (d) E must be an officer?
   (e) F will serve only if he is president?
5. A child has 12 blocks, of which 6 are red, 3 are green, 2 are blue, and 1 is black. If the child puts the blocks in a line, how many arrangements are possible?
6. If 4 Americans, 3 French people, and 3 British people are to be seated in a row.
   (a) How many seating arrangements are possible for these 10 individuals when there is no restriction on where each person may be seated?
   (b) How many seating arrangements are possible when people of the same nationality must be seated next to each other?
   (c) How many seating arrangements are possible when the 4 Americans must sit next to each other but there is no restriction on the others?
7. For years, telephone area codes in the United States and Canada consisted of a sequence of three digits. The first digit was an integer from 2 through 9; the second digit was either 0 or 1; and the third digit was any integer from 1 through 9.
   (a) How many area codes were possible?
   (b) How many area codes ending in 6 were possible?
8. In how many ways can 4 novels, 3 biology books, and 2 mathematics books be arranged on a bookshelf if
   (a) the books can be arranged in any order;
   (b) the biology books must be together and the novels must be together;
   (c) the novels must be together but the other books can be arranged in any order?
9. Two pollsters will canvas a neighborhood with 20 houses. Each pollster will visit 10 houses. No house will be visited more than once. How many different assignments of pollsters to houses are possible?
10. A box contains 24 light bulbs, of which two are defective. If a person selects 10 light bulbs at random, without replacement, what is the probability that both defective light bulbs will be selected?
11. Suppose that two defective refrigerators have been included in a shipment of six refrigerators. Now suppose that the buyer begins to test the refrigerators one at a time.
    (a) What is the probability that the last defective refrigerator is found on the fourth test?
    (b) What is the probability that no more than four refrigerators need to be tested before both defective refrigerators are found?

12. Consider two collections of 4 cards, one that contains 4 red cards numbered from 1 to 4 and one that contains 4 blue cards numbered from 1 to 4. Suppose that one card is selected at random from the 4 red cards and one card is selected at random from the 4 blue cards. Let A denote the event that the number on the red card is larger than the number on the blue card. Let B denote the event that the number on the red card is greater than two. Let C denote the event that the number on the blue card is odd.

    (a) List the 16 possible outcomes of this experiment as ordered pairs with the first element representing the number on the red card and the second element representing the number on the blue card, *e.g.* the ordered pair $(1, 4)$ indicates that the red card numbered 1 was selected and the blue card numbered 4 was selected.

    Using this collection of 16 ordered pairs as a representation of the sample space Ω, describe each of the following events both in words and as subsets of Ω:

    (b) $A \cap B$
    (c) $A \cup C$
    (d) $B \cap (A \cup C)$
    (e) $B^c \cap C^c$
    (f) $A \cap B \cap C$.

13. This problem is concerned with a simple visual messaging system based on an ordered arrangement of ten colored flags. A message is formed by placing ten colored flags on a single flagpole. The message is read by noting the colors of the flags starting at the top of the pole and moving down. Suppose that the ten flags consist of two red flags, three blue flags, and five green flags.

    (a) How many different messages can be formed from these ten flags?
    (b) How many different messages can be formed from these ten flags if the first and last flags must be red?
    (c) How many different messages can be formed from these ten flags if the fifth and sixth flags must be red?
    (d) How many different messages can be formed from these ten flags if the every other flag, starting with the first must be green?

14. Consider three groups, group A, group B, and group C, each consisting of ten people. A subgroup of three people is to be selected from the combined group of thirty people.

    (a) How many choices of three people are possible?
    (b) How many choices of three people are there in which all three people come from group A?
    (c) How many choices of three people are there in which all three people come from the same group of ten?
    (d) How many choices of three people are there in which one of the three people comes from group A, one comes from group B, and one comes from group C?

15. Suppose that five cards are selected at random without replacement from a standard deck of 52 playing cards.
    (a) Find the probability that the five cards selected include exactly three face cards (Jack, Queen, King).
    (b) Find the probability that the five cards selected include exactly three hearts and exactly two spades.
    (c) Find the probability that the five cards selected include exactly two clubs and exactly two diamonds.
    (d) Find the probability that the five cards selected consist of three sevens and two non–sevens which are not of the same kind.

16. A box contains 100 balls of which 20 are red, 30 are white, and 50 are blue. If 6 balls are selected at random with replacement from this box, find the following.
    (a) The probability that exactly 3 of the 6 balls are red
    (b) The probability that 2 of the 6 balls are red and 4 are white.
    (c) The probability that 1 of the 6 balls is red, 2 are white, and 3 are blue.

17. **This is the without replacement version of the preceding problem.** A box contains 100 balls of which 20 are red, 30 are white, and 50 are blue. If 6 balls are selected at random without replacement from this box, find the following.
    (a) The probability that exactly 3 of the 6 balls are red
    (b) The probability that 2 of the 6 balls are red and 4 are white.
    (c) The probability that 1 of the 6 balls is red, 2 are white, and 3 are blue.

18. Consider an elevator with five passengers. Suppose that this elevator stops at ten floors. Also assume that each passenger chooses the floor at which he or she gets off at random and independently of the other passengers. Find the probability that no two passengers get off at the same floor. **Hint:** You can think of the assignment of floors to passengers as the selection, at random and with replacement, of 5 numbers from the set $\{1, 2, \dots, 10\}$.

Chapter 6

Conditional Probability and Independence

6.4 Problems

1. You ask your neighbor to water a sickly plant while you are on vacation. Without water, it will die with probability .8; with water, it will die with probability .15. You are 90 percent certain that your neighbor will remember to water the plant.
   (a) What is the probability that the plant will be alive when you return?
   (b) If the plant is dead upon your return, what is the probability that your neighbor forgot to water it?
2. Box A contains 2 white balls and 1 black ball, whereas box B contains 1 white ball and 5 black balls. A ball is drawn at random from box A and transferred to box B. A ball is then drawn from the seven balls in box B. Given that the ball drawn from box B is white, what is the conditional probability that the ball transferred from box A to box B was white?
3. Maria will take two books with her on a trip. Suppose that the probability that she will like book 1 is .6, the probability that she will like book 2 is .5, and the probability that she will like both books is .4. Find the conditional probability that she will like book 2 given that she did not like book 1.
4. There is a 60 percent chance that event A will occur. If A does not occur, then there is a 10 percent chance that B will occur. What is the probability that at least one of the events A or B will occur?

5. Six balls are to be randomly chosen from a box containing 8 red, 10 green, and 12 blue balls. First assume that the balls are chosen at random with replacement.

   (a) What is the probability at least one red ball is chosen?
   (b) Given that no red balls are chosen, what is the conditional probability that there are exactly 2 green balls among the 6 chosen?

   Next assume that the balls are chosen at random without replacement.

   (c) What is the probability at least one red ball is chosen?
   (d) Given that no red balls are chosen, what is the conditional probability that there are exactly 2 green balls among the 6 chosen?

6. The probability that a new car battery functions for more than 3 years is .8, the probability that it functions for more than 4 years is .4, and the probability that it functions for more than 5 years miles is .1. If a new car battery is still working after 3 years, what is the probability that

   (a) its total life will exceed 4 years?
   (b) its additional life will exceed 2 years?

7. Two factories, A and B, produce radios. Each radio produced at factory A is defective with probability .05, whereas each one produced at factory B is defective with probability .01. Suppose you purchase two radios that were produced at the same factory, which is equally likely to have been either factory A or factory B. If the first radio is defective, what is the conditional probability that the other one is also defective?

8. Consider a game of bridge. First assume that West has no aces.

   (a) What is the conditional probability that his partner (East) has no aces?
   (b) What is the conditional probability that his partner (East) has 2 or more aces?

   Next assume that West has exactly 1 ace.

   (c) What is the conditional probability that his partner (East) has no aces?
   (d) What is the conditional probability that his partner (East) has 2 or more aces?

9. Suppose that one card is selected at random from a standard deck of 52 cards.

   (a) Let A denote the event that the card is a spade, and let B denote the event that the card is a king. Are A and B independent?
   (b) Let C denote the event that the card is red (a heart or diamond), and, as above, let B denote the event that the card is a king. Are C and B independent?
   (c) Let D denote the event that the card is a face card (a jack, queen, or king), and, as above, let B denote the event that the card is a king. Are D and B independent?

10. Suppose that a fair coin is tossed twice. Let A denote the event that the first toss results in a head, let B denote the event that the second toss results in a tail, let C denote the event that both tosses result in a head, and let D denote the event that the results of the two tosses are not the same.

    (a) Are A and B independent? Are A and C independent? Are A and D independent? Are B and C independent? Are B and D independent?
    (b) Are C and D independent?

11. Suppose that a balanced die is tossed twice. Let A denote the number observed on the first toss, let B denote the number observed on the second toss, and let C denote the sum of the numbers obtained on the two tosses.

    (a) Are the events $[A = 3]$ and $[C = 6]$ independent?
    (b) Are the events $[A = 3]$ and $[C = 7]$ independent?
    (c) Are the events $[B = 4]$ and $[C = 6]$ independent?
    (d) Are the events $[B = 4]$ and $[C = 7]$ independent?
    (e) Are the events $[A \text{ is even}]$ and $[B = 3]$ independent?
    (f) Are the events $[A \text{ is even}]$ and $[C = 4]$ independent?
    (g) Are the events $[A \text{ is even}]$ and $[B \text{ is even}]$ independent?
    (h) Are the events $[A = 4]$, $[B = 3]$, and $[C = 7]$ independent? Are they pairwise independent?

12. Consider two boxes, say box A and box B. Suppose that box A contains 60 red balls and 40 white balls while box B contains 30 red balls and 70 white balls.

    Suppose that a balanced die is tossed once. If the number on the die is one or two, then one ball is selected from box A and if it is three, four, five, or six, then one ball is selected from box B.

    (a) Find the probability that the ball selected is red.

    Now let's draw two balls from the box. As before, if the number on the die is one or two, we select from box A and if it is three, four, five, or six, then we select from box B. But this time, we select 2 balls at random with replacement.

    (b) Find the probability that both of the two balls selected are red.
    (c) Find the probability that exactly one of the two balls selected is red.

    Now let's turn the question around. Suppose that we have completed this experiment and selected the two balls.

    (d) Given that both of the two balls are red, what is the probability that these balls came from box A?
    (e) Now suppose that exactly one of the two balls is red, what is the probability that these balls came from box A?

13. Suppose that a free medical test for a certain disease is available. This test is 90% reliable in the following sense: If a person has the disease, there is a probability of .9 that the test will correctly indicate that the person has the disease (a true positive); whereas, if a person does not have the disease, there is a probability of .1 that the test will incorrectly indicate that the person has the disease (a false positive). In addition, suppose that this disease is rare in the sense that there is only a 1 in 10,000 chance that you have the disease. Suppose that, on a whim, you decide to take the test and when the results come back the test indicates that you have the disease! With this additional information how much does the probability that you actually have the disease increase? In other words, how does the conditional probability that you have the disease given that the test indicates that you have the disease compare to the unconditional probability that you have the disease?

14. Balls are selected at random and removed, one at a time, from a box that initially contains 20 red and 10 blue balls.

    (a) What is the probability that all of the red balls are removed before all of the blue ones have been removed?

    Now suppose that the box initially contains 20 red, 10 white, and 8 blue balls.

    (b) Now what is the probability that all of the red balls are removed before all of the white ones have been removed?
    (c) What is the probability that the colors are depleted in the order blue, white, red? That is, all red are removed first, then all white, and finally all blue.
    (d) What is the probability that the group of blue balls is the first of the three color groups to be removed?

Chapter 7

Discrete Random Variables

7.10 Problems

1. Let X denote a discrete random variable with probability mass function (p.m.f.)

$$p_X(x) = \begin{cases} 2c & \text{for } x = 1 \\ 4c & \text{for } x = 2 \\ 6c & \text{for } x = 3 \\ 8c & \text{for } x = 4 \\ 7c & \text{for } x = 5 \\ 5c & \text{for } x = 6 \\ 3c & \text{for } x = 7 \\ 1c & \text{for } x = 8 \\ 0 & \text{otherwise.} \end{cases}$$

   where c is a suitable constant. *Do not round off — express all values as rational fractions.*

   (a) Find the value of c for which this is a valid probability mass function.
   (b) Find the probability $P(X \leq 5)$.
   (c) Find the probability $P(2 < X \leq 5)$.
   (d) Find the expected value of X.
   (e) Find the expected value of X^2.
   (f) Find the variance of X.

2. Consider a box containing 3 red balls and 7 white balls. Suppose that balls are drawn one at a time, at random, without replacement from this box until two red balls are obtained. Let X denote the number of the draw on which the second red ball is obtained.

   (a) Find the probability mass function of X.
   (b) Find the expected value of X.
   (c) Find the expected value of X^2.
   (d) Find the variance of X.

3. An individual who has automobile insurance from a certain company is randomly selected. Let X be the number of moving violations for which the individual was cited during the last 3 years.

   According to their most recent records, the distribution of X, the number of moving violations for which its customers have been cited during the last 2 years, has p.m.f.

$$p_X(x) = \begin{cases} 0.60 & \text{if } x = 0 \\ 0.25 & \text{if } x = 1 \\ 0.10 & \text{if } x = 2 \\ 0.05 & \text{if } x = 3 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Find the expected number of such citations for a driver insured by this company.
   (b) Suppose this company charges an individual with X moving violations a surcharge of $Y = 100X^2$ dollars. Calculate the expected amount of the surcharge.

4. A certain brand of upright freezer is available in three different rated capacities: 16, 18, and 20 cubic feet. Let X denote the rated capacity of a freezer of this brand sold at a certain store. Suppose that X has p.m.f.

$$p_X(x) = \begin{cases} 0.2 & \text{if } x = 16 \\ 0.5 & \text{if } x = 18 \\ 0.3 & \text{if } x = 20 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Compute $E(X)$, $E(X^2)$, and $\text{var}(X)$.
   (b) If the price of a freezer having capacity X is $Y = 70X - 650$, what is the expected price paid by the next customer to buy a freezer?
   (c) What is the variance of the price paid by the next customer?
   (d) Suppose that although the rated capacity of a freezer is X, the actual capacity is $W = X - 0.008X^2$. What is the expected actual capacity of the freezer purchased by the next customer?

5. NBC News reported on May 2, 2013, that 1 in 20 children in the United States have a food allergy of some sort. Consider selecting a random sample of 25 children and let X be the number in the sample who have a food allergy. For the purposes of this exercise, assume that X follows the binomial distribution with $n = 25$ and $p = .05$.

   (a) Determine both $P(X \leq 3)$ and $P(X < 3)$
   (b) Determine $P(X \geq 4)$
   (c) Determine $P(1 \leq X \leq 3)$
   (d) Find $E(X)$ and $\mathrm{SD}(X) = \sigma_X$
   (e) In a sample of 50 children, what is the probability that none has a food allergy?

6. A particular telephone number is used to receive both voice calls and fax messages. Suppose that 25% of the incoming calls involve fax messages, and consider a random sample of 25 incoming calls.

   (a) Find the probability that at most 6 of the calls involve a fax message?
   (b) Find the probability that exactly 6 of the calls involve a fax message?
   (c) Find the probability that at least 6 of the calls involve a fax message?
   (d) Find the probability that more than 6 of the calls involve a fax message?

7. Suppose that 30% of all students who have to buy a physical textbook for a particular course choose a new copy, whereas the other 70% choose a used copy. Consider a random sample of 25 purchasers and let X denote the number of these purchasers who choose a new copy of the textbook.

   (a) Find the expected value and standard deviation of X.
   (b) What is the probability that the number who choose new copies is more than two standard deviations away from the mean value?
   (c) The bookstore has 15 new copies and 15 used copies in stock. If 25 people come to the bookstore to purchase this textbook, what is the probability that all 25 will get the type of book they want from current stock? *Hint: For what values of X will all 25 get what they want?*
   (d) Suppose that new copies cost \$100 and used copies cost \$70. Assume the bookstore currently has 50 new copies and 50 used copies. What is the expected value of total revenue from the sale of the next 25 copies purchased? Hint: You can express the total revenue as a linear function of X.

8. Eighteen individuals are scheduled to take a driving test at a particular DMV office on a certain day, eight of whom will be taking the test for the first time. Suppose that six of these individuals are randomly assigned to a particular examiner, and let X be the number among the six who are taking the test for the first time.

   (a) What is the name of the distribution of X?
   (b) Compute $P(X = 2)$, $P(X \leq 2)$, and $P(X \geq 2)$.
   (c) Find the expected value and standard deviation of X.

9. An instructor who taught two sections of a particular course last semester, section A with 30 students and section B with 20, assigned a term project. After all 50 projects had been turned in, the instructor randomly ordered them before grading. Consider the first 15 projects in this random ordering. Let X denote the number of these 15 projects that are from section A.
   (a) Find the probability that exactly 10 of these 15 projects are from section A.
   (b) Find the probability that at least 10 of these 15 projects are from section A.
   (c) Find the probability that at least 10 of these 15 projects are from the same section.
   (d) Find the expected value and standard deviation of the number among these 15 that are from the section A.
   (e) Find the expected value and standard deviation of the number of projects not among these first 15 that are from the section A.

10. A personnel director interviewing 11 applicants for four job openings has scheduled six interviews for the first day and five for the second day of interviewing. Assume that the candidates are interviewed in random order. Let X be the number of the top 4 candidates interviewed on the first day.
   (a) What is $P(X = x)$, the probability that x of the top four candidates are interviewed on the first day? Be sure to indicate the values of x for which this probability is not zero.
   (b) How many of the top four candidates can be expected to be interviewed on the first day?

11. A recent news report stated that, 1 in 20 children in the United States has a food allergy of some sort. Suppose a random sample of 25 children is selected and let X denote the number of children in the sample who have a food allergy.
   (a) Find the probability that no more than 3 of these children have a food allergy, $P(X \leq 3)$.
   (b) Find the probability that less than 3 of these children have a food allergy, $P(X < 3)$.
   (c) Find the probability that at least 4 of these children have a food allergy, $P(X \geq 4)$.
   (d) Find the probability that between 1 and 3, inclusive, of these children have a food allergy, $P(1 \leq X \leq 3)$.
   (e) Find the expected value of X, $E(X)$.
   (f) Find the variance of X, $\text{var}(X)$, and the standard deviation of X, $\text{SD}(X)$.
   (g) Now suppose we take a sample of 50 children. What is the probability that none of the 50 children has a food allergy?

12. Assume that the probability that a letter will be delivered within three working days is .9. Suppose that 10 letters inviting friends for a dinner party are sent out on Tuesday. Everyone who receives the invitation by Friday (*i.e.*, within 3 working days) will come. Those who do not receive the invitation by Friday will not come. Let X denote the number of friends who come to dinner.

    (a) What is the name of the distribution of X?
    (b) Find the probability that at least 7 friends will come.
    (c) What are the expected value and variance of X.
    (d) If the caterer charges a base fee of \$100 plus \$10 for each guest who comes to the dinner party. What is the expected value and variance of the catering cost?

13. A jewelry dealer is considering the sale of an antique necklace. She estimates that there is a 20% chance that she will make a profit of \$250 on the sale, a 40% chance that she will make a profit of \$150 on the sale, a 30% chance that she will break even on the sale, and a 10% chance that she will lose \$50 on the sale. What is her expected profit?
14. A manufacturer ships parts in lots of 1000 and makes a profit of \$50 per lot sold. The purchaser, however, subjects the product to a sampling plan as follows: 10 parts are selected at random with replacement. If none of these parts are defective, the lot is purchased; if one part is defective, the lot is purchased but the manufacturer returns \$10 to the buyer; if two or more parts are found to be defective, the entire lot is returned at a net loss of \$25 to the manufacturer. What is the manufacturer's expected profit if 10% of the parts are defective?
15. A jury trial ended in a hung jury with 8 jurors voting guilty and 4 jurors voting for acquittal. Suppose that the 12 jurors exit the courtroom in random order. How many of the first four jurors to leave the courtroom would be expected to have voted for acquittal? In other words, what is the expected value of X, when X denotes the number of jurors out of the first four jurors to leave the courtroom who voted for acquittal?
16. A salesman has scheduled two sales appointments. The probability that he will be able to close the deal at his first appointment is .75. The probability that he will be able to close the deal at his second appointment is .45. If he closes the deal at the first appointment he will earn a commission of \$1,000 on that sale and, regardless of the outcome of the first appointment, if he closes the deal at the second appointment he will earn a commission of \$1,500 on that sale. Assuming that the outcomes of the deals at the two appointments are independent, what is the salesman's expected profit?

Chapter 8

Continuous Random Variables

8.5 Problems

Instructions

For each of the questions below:

1. Define the random variable. That is, identify the quantity being modeled and define the corresponding random variable. For example, in the NHANES example in the book we measured the heights of a random sample of adult women in the United States so we would say let X denote the height of an adult woman in the United States. Then express the answer in symbolic form as a function of X. For example, for X as defined above, the probability that the height of a randomly selected adult woman in the United States exceeds 60 inches can be expressed as $P(X > 60)$.
2. Sketch a normal density curve, label the x–axis with the value of the mean and the values relevant for the question, and shade the relevant area or areas under the density curve.
3. Compute the requested numerical value or values to 4 decimal place accuracy.

You can use GeoGebra to do normal probability calculations and to create normal density curves.

You can also use a TI–84 (and similar) calculator to do normal probability calculations.

`normalcdf`$(a, b, \mu, \sigma) = P(a < X \leq b)$ when X follows a normal distribution with mean μ and standard deviation σ.

`invNorm`(p, μ, σ) gives the value of a such that $P(X \leq a) = p$ when X follows a normal distribution with mean μ and standard deviation σ.

When a problem asks how would you characterize the largest 5% of the observations, it is asking you to determine the value of a for which $P(X \geq a) = .05$

Problems

1. The birth weight of babies born in the normal range of 37–43 weeks gestational age in the US can be reasonably modeled using a normal distribution with mean 7.7 pounds and standard deviation 1.3 pounds.

   (a) Find the probability that the birth weight of a randomly selected baby of this type exceeds 9.1 pounds.

   (b) Find the probability that the birth weight of a randomly selected baby of this type is between 6.5 and 9.1 pounds.

   (c) Find the probability that the birth weight of a randomly selected baby of this type is either less than 6 pounds or greater than 9 pounds.

   (d) How would you characterize the largest 5% of all such birth weights?

   (e) How would you characterize the smallest 10% of all such birth weights?

2. A machine operation produces bearings whose diameters can be reasonably modeled as following a normal distribution with mean $\mu = 0.500$ inches and standard deviation $\sigma = 0.002$ inches.

   (a) Find the probability that the diameter of a randomly selected bearing of this type has an actual diameter of most .504 inches.

   (b) If specifications require that the bearing diameter be within .004 inches of .500, what fraction of the production will be unacceptable?

   (c) How would you characterize the largest 5% of the actual diameters of all the bearings produced using this machine?

   (d) Using .95 (95%) as a "large proportion of the time", find the range of actual diameters of all the bearings produced using this machine with the property that a value in this range will be selected with probability .95. That is, the range of values with the property that 95% of the bearings have diameters within this range.

3. A machine was designed to fill 12 ounce cups with a beverage. The target fill amount is exactly 12 oz and the acceptable fill range is 11.75 to 12.25 ounces. The amount of beverage dispensed by this machine can be reasonably modeled using a normal distribution with mean $\mu = 12$ oz and standard deviation $\sigma = 0.13$ oz.

   (a) Find the probability that the amount of beverage dispensed by this machine will be within the specified acceptable fill range.

   (b) If the machine dispenses more than 12.35 oz, then the cup will overflow wasting beverage and contaminating the outside of the cup. Find the probability that this machine will overfill the cup.

   (c) If the fill level in the cup is below 11.75 ounces, the customer will perceive the cup as being under-filled. Find the probability that the amount of beverage dispensed by the machine will be low enough to be perceived as under-filled by such a customer.

   (d) Using .95 (95%) as a "large proportion of the time", find the range of fill values produced using this machine with the property that a value in this range will occur with probability .95. That is, the range of values with the property that 95% of time the amount of beverage dispensed by this machine will be within this range.

4. The blood cholesterol levels of middle aged men in the US can be reasonably modeled using a normal distribution with mean $\mu = 222$ mg/dl and standard deviation $\sigma = 36$ mg/dl.

   (a) Find the probability that the blood cholesterol level of a randomly selected man of this type would be considered high in the sense of exceeding 240 mg/dl.

   (b) Find the probability that the blood cholesterol level of a randomly selected man of this type would be considered elevated in the sense of falling in the range from 200 mg/dl to 240 mg/dl.

   (c) Find the probability that the blood cholesterol level of a randomly selected man of this type is either less than 186 mg/dl or greater than 312 mg/dl.

   (d) How would you characterize the largest 10% of all such blood cholesterol levels?

   (e) How would you characterize the smallest 5% of all such blood cholesterol levels?

5. A gasoline tank for a certain car is designed to hold 15 gallon of gas. We can reasonably model the distribution of the actual usable capacity of the tank using a normal distribution with mean 15 gallons and standard deviation 0.1 gallons.

   (a) Find the probability that the gas tank of a randomly selected car of this type has an actual usable capacity of at most 15.2 gallons.

   (b) Find the probability that the gas tank of a randomly selected car of this type has an actual usable capacity of between 14.7 and 15.1 gallons.

   (c) How would you characterize the largest 5% of the actual usable capacities of the gas tanks of all such cars?

   (d) Using .95 (95%) as a "large proportion of the time", find the range of actual usable capacity of gas tanks of cars of this type with the property that a value in this range will be selected with probability .95. That is, the range of values with the property that 95% of the gas tanks of all such cars have actual usable capacities within this range.