8.5 Problems (last update 1 November 2024)

For each of the questions below:

i) Define the random variable. That is, identify the quantity being modeled and define the corresponding random variable. For example, in the NHANES example in the book we measured the heights of a random sample of adult women in the United States so we would say let X denote the height of an adult woman in the United States. Then express the answer in symbolic form as a function of X. For example, the probability that the height of a randomly selected adult woman in the United States exceeds 60 inches can be expressed as $P(X > 60)$.

ii) Sketch a normal density curve, label the X–axis with the value of the mean and the values relevant for the question, and shade the relevant area or areas under the density curve.

iii) Compute the requested numerical value or values to 4 decimal place accuracy.

You can use GeoGebra to do normal probability calculations and to create normal density curves.

You can also use a TI–84 (and similar) calculator to do normal probability calculations.
`normalcdf`$(a, b, \mu, \sigma) = P(a < X \leq b)$ when X follows a normal distribution with mean μ and standard deviation σ.
`invNorm`(p, μ, σ) gives the value of a such that $P(X \leq a) = p$ when X follows a normal distribution with mean μ and standard deviation σ.

When a problem asks how would you characterize the largest 5% of the observations, it is asking you to determine the value of a for which $P(X \geq a) = .05$

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1. The birth weight of babies born in the normal range of 37–43 weeks gestational age in the US can be reasonably modeled using a normal distribution with mean 7.7 pounds and standard deviation 1.3 pounds.

a) Find the probability that the birth weight of a randomly selected baby of this type exceeds 9.1 pounds.
b) Find the probability that the birth weight of a randomly selected baby of this type is between 6.5 and 9.1 pounds.
c) Find the probability that the birth weight of a randomly selected baby of this type is either less than 6 pounds or greater than 9 pounds.
d) How would you characterize the largest 5% of all such birth weights?
e) How would you characterize the smallest 10% of all such birth weights?

2. A machine operation produces bearings whose diameters can be reasonably modeled as following a normal distribution with mean $\mu = .500$ inches and standard deviation $\sigma = .002$ inches.

a) Find the probability that the diameter of a randomly selected bearing of this type has an actual diameter of most .504 inches.
b) If specifications require that the bearing diameter be within .004 inches of .500, what fraction of the production will be unacceptable?
c) How would you characterize the largest 5% of the actual diameters of all the bearings produced using this machine?
d) Using .95 (95%) as a "large proportion of the time", find the range of actual diameters of all the bearings produced using this machine with the property that a value in this range will be selected with probability .95. That is, the range of values with the property that 95% of the bearings have diameters within this range.

3. A machine was designed to fill 12 ounce cups with a beverage. The target fill amount is exactly 12 oz and the acceptable fill range is 11.75 to 12.25 ounces. The amount of beverage dispensed by this machine can be reasonably modeled using a normal distribution with mean $\mu = 12$ oz and standard deviation $\sigma = .13$ oz.

a) Find the probability that the amount of beverage dispensed by this machine will be within the specified acceptable fill range.

b) If the machine dispenses more than 12.35 oz, then the cup will overflow wasting beverage and contaminating the outside of the cup. Find the probability that this machine will overfill the cup.

c) If the fill level in the cup is below 11.75 ounces, the customer will perceive the cup as being underfilled. Find the probability that the amount of beverage dispensed by the machine will be low enough to be perceived as underfilled by such a customer.

d) Using .95 (95%) as a "large proportion of the time", find the range of fill values produced using this machine with the property that a value in this range will occur with probability .95. That is, the range of values with the property that 95% of time the amount of beverage dispensed by this machine will be within this range.

4. The blood cholesterol levels of middle aged men in the US can be reasonably modeled using a normal distribution with mean $\mu = 222$ mg/dl and standard deviation $\sigma = 36$ mg/dl.

a) Find the probability that the blood cholesterol level of a randomly selected man of this type would be considered high in the sense of exceeding 240 mg/dl.

b) Find the probability that the blood cholesterol level of a randomly selected man of this type would be considered elevated in the sense of falling in the range from 200 mg/dl to 240 mg/dl.

c) Find the probability that the blood cholesterol level of a randomly selected man of this type is either less than 186 mg/dl or greater than 312 mg/dl.

d) How would you characterize the largest 10% of all such blood cholesterol levels?

e) How would you characterize the smallest 5% of all such blood cholesterol levels?