Math 270–005: Calculus I
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Homework 10
SOLUTIONS

§4.7

24. This is an $\frac{\infty}{\infty}$ indeterminate. We can divide both numerator and denominator by x^3, or we can use L'Hopital's Rule. We have:

$$\begin{aligned}\lim_{x\to\infty}\frac{4x^3-2x^2+6}{\pi x^3+4} &\stackrel{\text{LH}}{=} \lim_{x\to\infty}\frac{12x^2-4x}{3\pi x^2} = \lim_{x\to\infty}\left(\frac{4}{\pi}-\frac{4}{3\pi x}\right)\\ &= \frac{4}{\pi}-0 = \frac{4}{\pi}.\end{aligned}$$

You could also apply L'Hopital's Rule twice momre to $\frac{12x^2-4x}{3\pi x^2}$ instead.

35. This is a $\frac{0}{0}$ indeterminate, and applying L'Hopital's Rule leads to another $\frac{0}{0}$ indeterminate, so we do it yet again. We have:

$$\begin{aligned}\lim_{x\to\pi}\frac{\cos x+1}{(x-\pi)^2} &\stackrel{\text{LH}}{=} \lim_{x\to\pi}\frac{-\sin x}{2(x-\pi)}\\ &\stackrel{\text{LH}}{=} \lim_{x\to\pi}\frac{-\cos x}{2} = \frac{1}{2}.\end{aligned}$$

53. This is an $\infty\times 0$ indeterminate. We rewrite and apply L'Hopital's Rule to the resulting $\frac{0}{0}$ indeterminate:

$$\begin{aligned}\lim_{x\to 0} x\csc x = \lim_{x\to 0} x\left(\frac{1}{\sin x}\right) &= \lim_{x\to 0}\frac{x}{\sin x}\\ &\stackrel{\text{LH}}{=} \lim_{x\to 0}\frac{1}{\cos x} = \frac{1}{1} = 1.\end{aligned}$$

59. This is an $\infty-\infty$ indeterminate. We do the difference to turn it into a $\frac{0}{0}$ indeterminate. Then use L'Hopital's Rule again:

$$\begin{aligned}\lim_{x\to 1^+}\left(\frac{1}{x-1}-\frac{1}{\ln x}\right) &= \lim_{x\to 1^+}\frac{\ln x - x + 1}{(x-1)\ln x} \stackrel{\text{LH}}{=} \lim_{x\to 1^+}\frac{\frac{1}{x}-1}{\ln x + (x-1)\frac{1}{x}}\\ &= \lim_{x\to 1^+}\frac{\frac{1}{x}-1}{\ln x + 1 - \frac{1}{x}} \stackrel{\text{LH}}{=} \lim_{x\to 1^+}\frac{-\frac{1}{x^2}}{\frac{1}{x}+\frac{1}{x^2}}\\ &= \lim_{x\to 1^+}\frac{-1}{x^2\left(\frac{1}{x}+\frac{1}{x^2}\right)} = \lim_{x\to 1^+}\frac{-1}{x+1}\\ &= \frac{-1}{1+1} = -\frac{1}{2}.\end{aligned}$$