Math 270–005: Calculus I
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Homework 11
SOLUTIONS

§5.1

9. See the book for the graph. The length of each subinterval is $\Delta x = \frac{7-1}{6} = 1$. The values of the function at the grid points are

$$f(1) = 10,\ f(2) = 9,\ f(3) = 7,\ f(4) = 5,\ f(5) = 2,\ f(6) = 1\, f(7) = 0.$$

So the left Riemann sum is

$$f(1)(1) + f(2)(1) + f(3)(1) + f(4)(1) + f(5)(1) + f(6)(1) = 10 + 9 + 7 + 5 + 2 + 1 = 34.$$

The right Riemann sum is

$$f(2)(1) + f(3)(1) + f(4)(1) + f(5)(1) + f(6)(1) + f(7)(1) = 9 + 7 + 5 + 2 + 1 + 0 = 24.$$

15. See the book for the graphs:

(a) The function is $v = 3t^2 + 1$. The midpoints are $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$. So the approximation is:

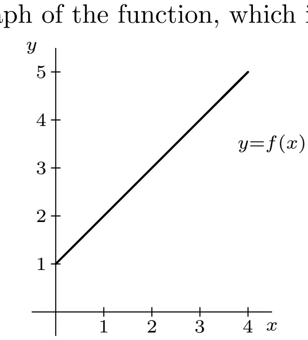
$$v(.5) + v(1.5) + v(2.5) + v(3.5) = 1.75 + 7.75 + 19.75 + 37.75 = 67 \text{ ft}.$$

(b) With the further divisions, now the interval length is $\frac{1}{2}$, and the midpoints are $\frac{1}{4}, \frac{3}{4}, \ldots, \frac{15}{4}$. So we have:

$$\begin{aligned} &v(0.25)(0.5) + v(0.75)(0.5) + v(1.25)(0.5) + v(1.75)(0.5)\\ &\qquad + v(2.25)(0.5) + v(2.75)(0.5) + v(3.25)(0.5) + v(3.75)(0.5)\\ &\qquad = 0.5(1.1875 + 2.6875 + 5.6875 + 10.1875 + 16.1875 + 23.6875 + 32.6875 + 43.1875)\\ &\qquad = 0.5(135.5) = 67.75 \text{ ft}. \end{aligned}$$

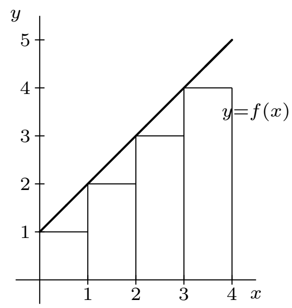
25. We have the function $f(x) = x + 1$ on the interval $[0, 4]$, and we want to divide the interval into $n = 4$ subintervals.

(a) First we want a sketch of the graph of the function, which is a straight line:

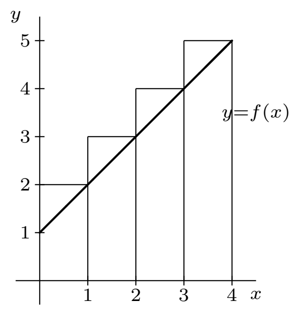


(b) Here $\Delta x = \frac{4-0}{4} = 1$, and the grid points are $x_0 = 0$, $x_1 = 1$, $x_2 = 2$, $x_3 = 3$ and $x_4 = 4$.

(c) The left Riemann sum is given by adding up the following rectangles:



From the picture, we see that this is an underestimate.
The right Riemann sum is given by adding the following rectangles:



From the picture we can see that this is an overestimate.

(d) The left Riemann sum is given by:

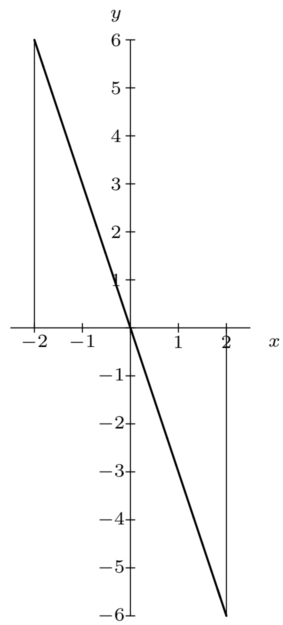
$$\sum_{k=1}^{4} f(x_k^*)\Delta x = \sum_{k=1}^{4} f(x_{k-1})(1)$$
$$= f(0) + f(1) + f(2) + f(3) = 1 + 2 + 3 + 4 = 10.$$

The right Riemann sum is given by

$$\sum_{k=1}^{4} f(x_k^*)\Delta x = \sum_{k=1}^{4} f(x_k)(1)$$
$$= f(1) + f(2) + f(3) + f(4) = 2 + 3 + 4 + 5 = 14.$$

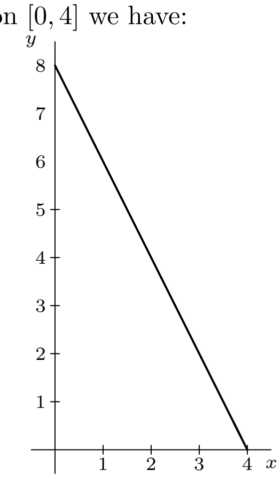
§5.2

27. We want the region between the graph of $y = -3x$ and the x-axis, for $-2 \leq x \leq 2$:



So the area of the region is 0, with the left "positive" triangle canceling out the right "negative" triangle.

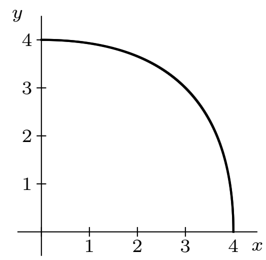
39. Graphing the function $y = 8 - 2x$ on $[0, 4]$ we have:



This region has an area of $\frac{1}{2}(8)(4) = 16$. So

$$\int_0^4 (8 - 2x)\, dx = 16.$$

43. The graph of $y = \sqrt{16 - x^2}\, dx$ is the top half of the circle with radius 4 centered at the origin:



So the area is one quarter the area of a circle of radius 4:

$$\int_0^4 \sqrt{16-x^2}\,dx = \frac{1}{4}\pi(4^2) = 4\pi.$$

47. See the book for the picture related to this problem and the next two.

    From the picture, the region between 0 and π consists of regions 1 and 2, which have areas 1 and $\pi - 1$, respectively. So, writing $A(R_i)$ for the area of region i, we have:

$$\int_0^\pi x\sin x\,dx = A(R_1) + A(R_2) = 1 + (\pi - 1) = \pi.$$

48. Here we have the areas of regions 1 and 2, minus the area of region 3 which is $\pi + 1$. So we have

$$\int_0^{3\pi/2} x\sin x\,dx = A(R_1) + A(R_2) - A(R_3) = 1 + (\pi - 1) - (\pi + 1) = -1.$$

49. For the integral up to 2π we also need to take the areas of regions 3 and 4, which are $\pi + 1$ and $2\pi - 1$, respectively; but because they are under the x-axis, they count as negative. We have:

$$\int_0^{2\pi} x\sin x\,dx = A(R_1) + A(R_2) - A(R_3) - A(R_4) = 1 + (\pi - 1) - (\pi + 1) - (2\pi - 1) = -2\pi.$$

50. For the integral from $\pi/2$ to 2π, we need the areas of regions 2 (positive), 3 (negative), and 4 (negative). We have:

$$\int_{\pi/2}^{2\pi} x\sin x\,dx = A(R_2) - A(R_3) - A(R_4) = (\pi - 1) - (\pi + 1) - (2\pi - 1) = -2\pi - 1.$$

51. Using the fact that $\int_0^4 3x(4-x)\,dx = 32$ and no other information, we can answer three of the following four integrals:

    (a) $\int_4^0 3x(4-x)\,dx = -\int_0^4 3x(4-x)\,dx = -32.$

    (b) $\int_0^4 x(x-4)\,dx = \int_0^4(-\frac{1}{3})(3x(4-x))\,dx = -\frac{1}{3}\int_0^4 3x(4-x)\,dx = -\frac{1}{3}(32) = -\frac{32}{3}.$

    (c) $\int_4^0 6x(4-x)\,dx = 2\int_4^0 3x(4-x)\,dx = -2\int_0^4 3x(4-x)\,dx = -2(32) = -64.$

    (d) We cannot evaluate $\int_0^8 3x(4-x)\,dx$ knowing only the integral from 0 to 4. We know that

$$\int_0^8 3x(4-x)\,dx = \int_0^4 3x(4-x)\,dx + \int_4^8 3x(4-x)\,dx = 32 + \int_4^8 3x(4-x)\,dx,$$

    but at that point we need additional information or calculations.

53. If $\int_0^3 f(x)\,dx = 2$, $\int_3^6 f(x)\,dx = -5$, and $\int_3^6 g(x)\,dx = 1$, then:

(a) $\int_0^3 5f(x)\,dx = 5\int_0^3 f(x)\,dx = 5(2) = 10.$

(b) $\int_3^6 (-3g(x))\,dx = -3\int_3^6 g(x)\,dx = -3(1) = -3.$

(c) $\int_3^6 (3f(x) - g(x))\,dx = 3\int_3^6 f(x)\,dx - \int_3^6 g(x)\,dx = 3(-5) - (1) = -16.$

(d) $\int_6^3 (f(x) + 2g(x))\,dx = \int_6^3 f(x) + 3\int_6^3 g(x)\,dx = -\int_3^6 f(x) - 2\int_3^6 g(x)\,dx = -(-5) - 2(1) = 3.$

§5.3

29. Since $(x^4)' = 4x^3$, we have

$$\int_0^2 4x^3\,dx = x^4 \Big|_0^2 = 16 - 0 = 16.$$

33. An antiderivative for $x + \sqrt{x}$ is $\frac{1}{2}x^2 + \frac{2}{3}x^{3/2}$, so

$$\int_0^1 (x + \sqrt{x})\,dx = \left(\frac{1}{2}x^2 + \frac{2}{3}x^{3/2}\right)\Big|_0^1 = \left(\frac{1}{2} + \frac{2}{3}\right) - (0 + 0) = \frac{7}{6}.$$

39. An antiderivative of $t^{-3} - 8$ is $-\frac{1}{2}t^{-2} - 8t$, so

$$\int_{1/2}^1 (t^{-3} - 8)\,dt = \left(-\frac{1}{2}t^{-2} - 8t\right)\Big|_{1/2}^1 = \left(-\frac{1}{2} - 8\right) - \left(-\frac{4}{2} - 4\right) = -\frac{1}{2} - 8 + 2 + 4 = -\frac{5}{2}.$$

45. Here,

$$\int_0^{\pi/4} \sec^2\theta\,d\theta = \tan\theta \Big|_0^{\pi/4} = \tan(\pi/4) - \tan(0) = 1 - 0 = 1.$$