

Math 272: Honors Calculus I

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Homework 1

SOLUTIONS

§2.2

5. See the book for the graph.

- (a) From the graph, $f(1) = -1$.
- (b) On the other hand, the graph suggests that $\lim_{x \rightarrow 1} f(x) = 1$, since the values of $f(x)$ approach 1 as x approaches 1.
- (c) The graph indicates that $f(0) = 2$.
- (d) Here, we also have $\lim_{x \rightarrow 0} f(x) = 2$, as the values of $f(x)$ are also approaching 2 as x approaches 0.

15. Please see the book for the graph.

- (a) From the graph, $f(1) = 0$.
- (b) The limit of $f(x)$ as x approaches 1 from the left is 1, $\lim_{x \rightarrow 1^-} f(x) = 1$, as seen in the graph.
- (c) The limit from the right is 0, though: $\lim_{x \rightarrow 1^+} f(x) = 0$.
- (d) Because the one-sided limits do not agree, we conclude that $\lim_{x \rightarrow 1} f(x)$ does not exist.

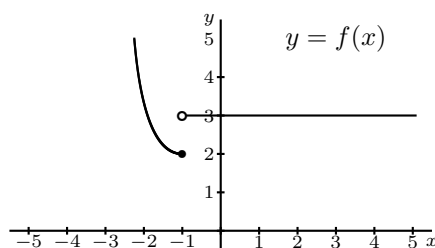
17. Please see the book for the graph.

- (a) $f(1) = 3$, the dot on the graph above $x = 1$.
- (b) $\lim_{x \rightarrow 1^-} f(x) = 2$.
- (c) $\lim_{x \rightarrow 1^+} f(x) = 2$.
- (d) $\lim_{x \rightarrow 1} f(x) = 2$.
- (e) $f(3) = 2$.
- (f) $\lim_{x \rightarrow 3^-} f(x) = 4$.
- (g) $\lim_{x \rightarrow 3^+} f(x) = 1$.
- (h) $\lim_{x \rightarrow 3} f(x)$ does not exist.
- (i) $f(2) = 3$.
- (j) $\lim_{x \rightarrow 2^-} f(x) = 3$.
- (k) $\lim_{x \rightarrow 2^+} f(x) = 3$.
- (l) $\lim_{x \rightarrow 2} f(x) = 3$.

19. The graph of

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x \leq -1, \\ 3 & \text{if } x > -1, \end{cases}$$

looks like the graph of $f(x) = x^2 + 1$ (the standard parabola $y = x^2$ raised by one unit) up to $x = -1$, and then is just a horizontal line at height $x = 3$.



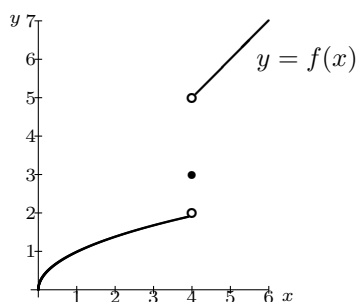
At $a = -1$, we see that $f(a) = f(-1) = (-1)^2 + 1 = 2$. On the other hand, $\lim_{x \rightarrow -1^-} f(x) = 2$, and $\lim_{x \rightarrow -1^+} f(x) = 3$. It is also evident from the graph that $\lim_{x \rightarrow -1} f(x)$ does not exist.

21. The function is

$$f(x) = \begin{cases} \sqrt{x} & \text{if } x < 4, \\ 3 & \text{if } x = 4, \\ x + 1 & \text{if } x > 4 \end{cases}$$

and we want to find the limits at $a = 4$. Note that the function is only defined for $x \geq 0$, though because we are looking for the limits at $a = 4$, the fact that it is not defined at negative values of x does not matter.

Here is a sketch of the graph:

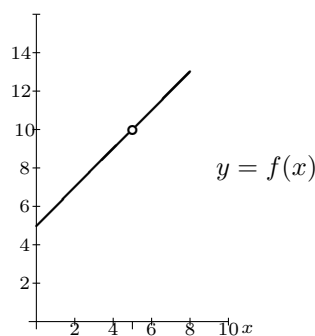


So $\lim_{x \rightarrow 4^-} f(x) = 2$, $\lim_{x \rightarrow 4^+} f(x) = 5$, and since they are different from each other, $\lim_{x \rightarrow 4} f(x)$ does not exist.

23. Here we are working with the function

$$f(x) = \frac{x^2 - 25}{x - 5},$$

at the point $a = 5$. Note that $x^2 - 25 = (x - 5)(x + 5)$, so for $x \neq 5$, this function takes the same values as the function $g(x) = x + 5$. Thus, we have:



So $\lim_{x \rightarrow 5^-} f(x) = 10$, $\lim_{x \rightarrow 5^+} f(x) = 10$, and therefore $\lim_{x \rightarrow 5} f(x) = 10$.

51. See the book for the graph of $\lfloor x \rfloor$.

(a) From the graph, we see:

$$\begin{aligned}\lim_{x \rightarrow -1^-} \lfloor x \rfloor &= -2 \\ \lim_{x \rightarrow -1^+} \lfloor x \rfloor &= -1 \\ \lim_{x \rightarrow 2^-} \lfloor x \rfloor &= 1 \\ \lim_{x \rightarrow 2^+} \lfloor x \rfloor &= 2.\end{aligned}$$

(b) Here we have

$$\begin{aligned}\lim_{x \rightarrow 2.3^-} \lfloor x \rfloor &= 2 \\ \lim_{x \rightarrow 2.3^+} \lfloor x \rfloor &= 2 \\ \lim_{x \rightarrow 2.3} \lfloor x \rfloor &= 2.\end{aligned}$$

(c) If a is an integer, then $\lim_{x \rightarrow a^-} \lfloor x \rfloor = a - 1$, and $\lim_{x \rightarrow a^+} \lfloor x \rfloor = a$.

(d) If a is not an integer, then there is a unique integer z such that $z < a < z + 1$. Then

$$\lim_{x \rightarrow a^-} \lfloor x \rfloor = \lim_{x \rightarrow a^+} \lfloor x \rfloor = z.$$

When $a > 0$, if we write a in decimal notation, $a = m.a_1a_2a_3\cdots$, we will have $z = m$. If $a < 0$, and we write a in decimal notation, $a = -m.a_1a_2a_3$ with $m > 0$, we will have $z = m - 1$. For example, if $a = -2.5$, then $-3 < a < -2$, so $z = -3$.

(e) The limit $\lim_{x \rightarrow a} \lfloor x \rfloor$ exists precisely when a is **not** an integer. When a is an integer, the limit from the left and the limit from the right are different, so the limit does not exist.