

Math 272: Honors Calculus I

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Homework 5

SOLUTIONS

§3.3

19. If $y = x^5$, then $y' = 5x^4$.

25. If $h(t) = \frac{t^2}{2} + 1$, then $h'(t) = \frac{1}{2}(2t) = t$.

29. If $g(t) = 100t^2$ then $g'(t) = 200t$.

37. If $f(x) = 3e^x + 5x + 5$, then $f'(x) = 3e^x + 5$.

47. If $f(x) = (2x+1)(3x^2+2)$, then to find the derivative (without the product rule), we first expand:

$$\begin{aligned}f(x) &= (2x+1)(3x^2+2) \\&= 6x^3 + 4x + 3x^2 + 2 \\&= 6x^3 + 3x^2 + 4x + 2. \\f'(x) &= 18x^2 + 6x + 4.\end{aligned}$$

51. Here we have:

$$\begin{aligned}h(x) &= (x^2+1)^2 \\&= x^4 + 2x^2 + 1 \\h'(x) &= 4x^3 + 4x.\end{aligned}$$

57. Here,

$$\begin{aligned}g(w) &= \frac{e^{2w} + e^w}{e^w} \\&= \frac{e^w(e^w + 1)}{e^w} \\&= e^w + 1.\end{aligned}$$

So $g'(w) = e^w$.

70. Here we have $f(x) = 3x^2 + 5e^x$. We want to find $f'(x)$, $f''(x)$, and $f'''(x)$. We have:

$$\begin{aligned}f(x) &= 3x^2 + 5e^x \\f'(x) &= 6x + 5e^x \\f''(x) &= 6 + 5e^x \\f'''(x) &= 5e^x.\end{aligned}$$

§3.4

19. First we use the Product Rule, then we simplify:

$$\begin{aligned}f(x) &= 3x^4(2x^2-1) \\f'(x) &= (3x^4)'(2x^2-1) + 3x^4(2x^2-1)' \\&= 12x^3(2x^2-1) + 3x^4(4x) \\&= 24x^5 - 12x^3 + 12x^5 = 36x^5 - 12x^3.\end{aligned}$$

23. Here we have:

$$\begin{aligned}
 f(t) &= t^{5/3} e^t \\
 f'(t) &= (t^{5/3})' e^t + t^{5/3} (e^t)' \\
 &= \frac{5}{3} t^{\frac{5}{3}-1} e^t + t^{5/3} e^t \\
 &= \frac{5}{3} t^{2/3} e^t + t^{5/3} e^t \\
 &= t^{2/3} e^t \left(\frac{5}{3} + t \right).
 \end{aligned}$$

25. Using the quotient rule, we have:

$$\begin{aligned}
 f(x) &= \frac{e^x}{e^x + 1} \\
 f'(x) &= \frac{(e^x + 1)(e^x)' - e^x(e^x + 1)'}{(e^x + 1)^2} \\
 &= \frac{(e^x + 1)(e^x) - e^x(e^x)}{(e^x + 1)^2} \\
 &= \frac{e^{2x} + e^x - e^{2x}}{(e^x + 1)^2} \\
 &= \frac{e^x}{(e^x + 1)^2}.
 \end{aligned}$$

33. Product rule:

$$\begin{aligned}
 g(w) &= e^w (w^3 - 1) \\
 g'(w) &= (e^w)'(w^3 - 1) + e^w (w^3 - 1)' \\
 &= e^w (w^3 - 1) + e^w (3w^2) = e^w (w^3 + 3w^2 - 1).
 \end{aligned}$$

37. Quotient Rule:

$$\begin{aligned}
 g(x) &= \frac{e^x}{x^2 - 1} \\
 g'(x) &= \frac{(x^2 - 1)(e^x)' - e^x(x^2 - 1)'}{(x^2 - 1)^2} \\
 &= \frac{(x^2 - 1)e^x - e^x(2x)}{(x^2 - 1)^2} \\
 &= \frac{e^x(x^2 - 2x - 1)}{(x^2 - 1)^2}.
 \end{aligned}$$

47. A quotient rule that requires a product rule along the way:

$$\begin{aligned}
 h(x) &= \frac{xe^x}{x + 1} \\
 h'(x) &= \frac{(x + 1)(xe^x)' - (xe^x)(x + 1)'}{(x + 1)^2} \\
 &= \frac{(x + 1)((x)'e^x + x(e^x)') - xe^x}{(x + 1)^2}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{(x+1)(e^x + xe^x) - xe^x}{(x+1)^2} \\
&= \frac{xe^x + x^2e^x + e^x + xe^x - xe^x}{(x+1)^2} \\
&= \frac{(x^2 + x + 1)e^x}{(x+1)^2}
\end{aligned}$$

§3.5

11. We have

$$\lim_{x \rightarrow 0} \frac{\sin(3x)}{x} = \lim_{x \rightarrow 0} \frac{3 \sin(3x)}{3x} = 3 \lim_{x \rightarrow 0} \frac{\sin(3x)}{3x} = 3(1) = 3.$$

13. Similarly,

$$\lim_{x \rightarrow 0} \frac{\sin 7x}{\sin 3x} = \lim_{x \rightarrow 0} \frac{\frac{7 \sin 7x}{7x}}{\frac{3 \sin 3x}{3x}} = \frac{7}{3} \left(\lim_{x \rightarrow 0} \frac{\frac{\sin 7x}{7x}}{\frac{\sin 3x}{3x}} \right) = \frac{7}{3} \left(\frac{1}{1} \right) = \frac{7}{3}.$$

15. This time,

$$\begin{aligned}
\lim_{x \rightarrow 0} \frac{\tan 5x}{x} &= \lim_{x \rightarrow 0} \frac{\frac{\sin 5x}{\cos 5x}}{x} = \lim_{x \rightarrow 0} \frac{\sin 5x}{x \cos 5x} = \left(\lim_{x \rightarrow 0} \frac{\sin 5x}{x} \right) \left(\lim_{x \rightarrow 0} \frac{1}{\cos 5x} \right) \\
&= \left(\lim_{x \rightarrow 0} \frac{5 \sin 5x}{5x} \right) \left(\lim_{x \rightarrow 0} \frac{1}{\cos 5x} \right) = \left(5 \lim_{x \rightarrow 0} \frac{\sin 5x}{5x} \right) \left(\lim_{x \rightarrow 0} \frac{1}{\cos 5x} \right) \\
&= 5 \left(\frac{1}{1} \right) = 5.
\end{aligned}$$

23. If $y = \sin x + \cos x$, then $y' = \cos x - \sin x$.

25. If $y = e^{-x} \sin x = \frac{\sin x}{e^x}$, we have (using the quotient rule):

$$\begin{aligned}
y' &= \frac{e^x(\sin x)' - (\sin x)(e^x)'}{(e^x)^2} = \frac{e^x \cos x - e^x \sin x}{(e^x)^2} \\
&= \frac{e^x(\cos x - \sin x)}{(e^x)^2} = \frac{\cos x - \sin x}{e^x} = e^{-x}(\cos x - \sin x).
\end{aligned}$$

31. If $y = \sin x \cos x$, then

$$y' = (\sin x)' \cos x + \sin x(\cos x)' = \cos x \cos x - \sin x \sin x = \cos^2 x - \sin^2 x.$$

If you want to be fancy, you can also note that $\cos^2 x - \sin^2 x = \cos(2x)$.

33. If $y = \cos^2 x = \cos x \cos x$, then $y' = (\cos x)' \cos x + \cos x(\cos x)' = -2 \sin x \cos x$. If you want to be fancy, note that $2 \sin x \cos x = \sin(2x)$, so we could rewrite this as $y' = -\sin(2x)$.

35. If $y = w^2 \sin w + 2w \cos w - 2 \sin w$, then we have:

$$\begin{aligned}
y' &= (w^2 \sin w)' + 2(w \cos w)' - 2(\sin w)' \\
&= (w^2)' \sin w + w^2(\sin w)' + 2((w)' \cos w + w(\cos w)') - 2 \cos w \\
&= 2w \sin w + w^2 \cos w + 2(\cos w - w \sin w) - 2 \cos w \\
&= 2w \sin w + w^2 \cos w + 2 \cos w - 2w \sin w - 2 \cos w \\
&= w^2 \cos w.
\end{aligned}$$

39. If $y = \frac{\sin x}{1+\cos x}$, then

$$\begin{aligned} y' &= \left(\frac{\sin x}{1+\cos x} \right)' = \frac{(1+\cos x)(\sin x)' - \sin x(1+\cos x)'}{(1+\cos x)^2} \\ &= \frac{(1+\cos x)\cos x - \sin x(-\sin x)}{(1+\cos x)^2} = \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2} \\ &= \frac{\cos x + 1}{(1+\cos x)^2} = \frac{1}{1+\cos x}. \end{aligned}$$

43. If $y = \sec x + \csc x$, then $y' = \sec x \tan x - \csc x \cot x$.

45. If $y = e^x \csc x$, then

$$y' = (e^x)' \csc x + e^x (\csc x)' = e^x \csc x - e^x \csc x \cot x = e^x \csc x (1 - \cot x).$$

57. If $y = x \sin x$, then

$$\begin{aligned} y &= x \sin x \\ y' &= (x)' \sin x + x(\sin x)' = \sin x + x \cos x \\ y'' &= (\sin x)' + (x)' \cos x + x(\cos x)' \\ &= \cos x + \cos x - x \sin x \\ &= 2 \cos x - x \sin x. \end{aligned}$$

59. With $y = e^x \sin x$, we have:

$$\begin{aligned} y &= e^x \sin x \\ y' &= (e^x)' \sin x + e^x (\sin x)' = e^x \sin x + e^x \cos x \\ &= e^x (\sin x + \cos x), \\ y'' &= (e^x)' (\sin x + \cos x) + e^x (\sin x + \cos x)' \\ &= e^x (\sin x + \cos x) + e^x (\cos x - \sin x) \\ &= e^x (\sin x + \cos x + \cos x - \sin x) \\ &= e^x (2 \cos x) = 2e^x \cos x. \end{aligned}$$