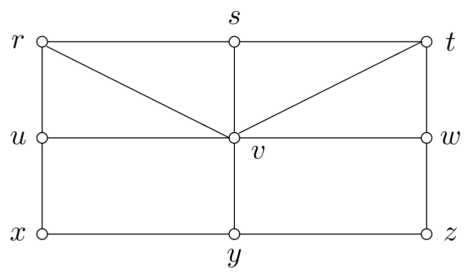
Math 483 - Spring 26

Homework 1

Due Thursday January 29

1. Let G be the graph below:



   Let e and f be the edges $e = ru$ and $f = vw$. Draw the subgraphs $G - \{e, f\}$ and $G - \{u, w\}$.

2. For the graph above, give an example of each of the following, or explain why no such example exists:

   (i) An x-y walk of length 6.
   (ii) A v-w trail that is not a v-w path.
   (iii) An r-z path of length 2.
   (iv) An x-z path of length 3.
   (v) An x-t path of length $d(x, t)$.
   (vi) A circuit of length 10.
   (vii) A cycle of length 8.
   (viii) A geodesic of length $\text{diam}(G)$.

3. Give an example of a connected graph G that includes three pairwise distinct vertices u, v, and w (and possibly other vertices), such that $d(u, v) = d(u, w) = d(v, w) = \text{diam}(G) = 3$.

4. We saw in class that a graph G of order at least 3 is connected if and only if G contains two distinct vertices u and v such that both $G - u$ and $G - v$ are connected. One might guess the following statement would also be true:

   Every connected graph G of order at least 4 contains three pairwise distinct vertices u, v, and w, such that each of $G - u$, $G - v$, and $G - w$ are connected.

   However, this statement is false. Draw a counterexample; that is, a graph G with at least 4 vertices, that is connected, and show that no choice of three distinct vertices will saisfy the condition.

5. Let G be a connected graph, and suppose that u and v are two distinct vertices in G. What is the minimum order of a connected subgraph of G that contains both u and v? Explain your answer.