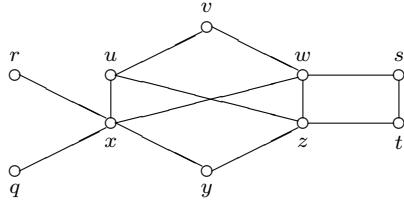


Math 483 - Spring 26

HOMEWORK 2

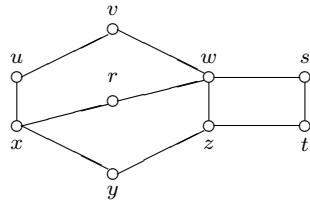
Due Thursday February 5

1. Let G be the following graph:



Determine whether G is bipartite or not. If it is bipartite, redraw it with vertices in two rows and all edges between the rows, clearly labeling the vertices. If it is not bipartite, explain why it is not bipartite.

2. Let G be the following graph:



Determine whether G is bipartite or not. If it is bipartite, redraw it with vertices in two rows and all edges between the rows, clearly labeling the vertices. If it is not bipartite, explain why it is not bipartite.

3. Let $G = K_5$ and $H = K_2$.

- (i) Draw $G \cup H$.
- (ii) Draw $G + H$.
- (iii) Draw $G \times H$.

4. Give an example of the following, or explain why no example exists.

- (i) A graph of order 7 with vertices of degree 1, 1, 1, 2, 2, 3, and 3.
- (ii) A graph of order 7 with vertices of degree 1, 2, 2, 2, 3, 3, and 7.
- 5. The degree of each vertex of a graph of order 12 and size 31 is either 4 or 6. How many vertices of degree 4 does the graph have?
- 6. Suppose that G is a disconnected graph that has exactly two odd vertices. Prove that the two odd vertices must be in the same connected component of G .
- 7. A graph G has the property that every edge of G joins an odd vertex with an even vertex. Prove that G is bipartite and has even size.
- 8. Let G be a connected graph that has the property that for every two distinct vertices u and v , either all $u-v$ paths have odd length, or else all $u-v$ paths have even length. Prove that G is bipartite.