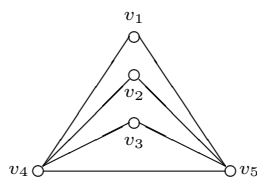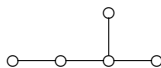# Math 483 - Spring 26
HOMEWORK 3
Due Thursday February 12

1. Let $r \geq 1$. Show that if G is both r-regular and bipartite with partite sets U and W, then $|U| = |W|$ (that is, U and W have the same number of vertices).

2. Let G be a graph of order n, and let $\overline{G}$ be its complement.

   (a) Prove that $\delta(G) + \delta(\overline{G}) \leq n - 1$.

   (b) Show that G is r-regular if and only if $\overline{G}$ is $(n - 1 - r)$-regular.

   (c) Prove that $\delta(G) + \delta(\overline{G}) = n - 1$ if and only if G is regular.

   (d) Prove that G is regular if and only if there is a vertex v of G such that $\deg_G(v) = \delta(G)$ and $\deg_{\overline{G}}(v) = \delta(\overline{G})$.

3. A *spanning subgraph* of a graph G is a subgraph H of G with $V(H) = V(G)$ (that is, H is obtained by removing edges from G, but keeping all the vertices).

   (a) Find spanning subgraphs G_r of the Petersen graph such that G_r is r-regular, for $r = 0, 1, 2,$ and 3.

   (b) Find induced subgraphs F_r of the Petersen graph such that F_r is r-regular, for $r = 0, 1, 2,$ and 3.

4. If the sequence $x, 7, 7, 5, 5, 4, 3, 2$ is graphical, what are the possible values of x, with $0 \leq x \leq 7$? Note that we need not have the resulting sequence be non-increasing.

5. Let G be the following graph:

   Find a 4-regular graph that contains G as an induced subgraph.

6. Let G be the graph from Problem 5.

   (a) Compute the adjacency matrix A of G.

   (b) Use the adjacency matrix to determine the number of length 3 walks from v_1 to v_5.

   (c) List all v_1-v_4 walks of length 3.

7. Let G be the graph from Problem 5. Label the edges and compute the incidence matrix of G.

8. Find a 3-regular graph of minimum order that contains the following graph H as an induced subgraph:

9. Use the Havel-Hakimi Theorem to determine if the given sequences are graphical or not:

   (a) s_1: $5, 3, 3, 3, 3, 2, 2, 2, 1.$

   (b) s_2: $6, 3, 3, 3, 3, 2, 2, 2, 2, 1, 1.$

   (c) s_3: $6, 5, 5, 4, 3, 2, 1.$