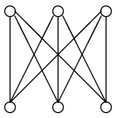
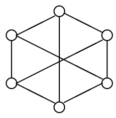
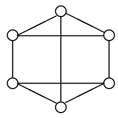
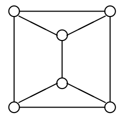
Math 483 - Spring 26

Homework 4

Due Thursday February 26

1. Give an example of three non-isomorphic graphs of order 5 and size 5.

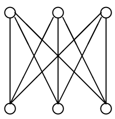
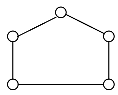
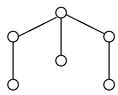
2. Below are four graphs. Which pairs of graphs are isomorphic, and which pairs are not? Justify your answer.

G_1:  G_2:  G_3:  G_4: 

3. Let G_1 and G_2 be two graphs, with vertex sets $V(G_1) = \{u_1, v_1, w_1, x_1, y_1, z_1\}$ and $V(G_2) = \{u_2, v_2, w_2, x_2, y_2, z_2\}$. If v_1 has degree 3 and is adjacent to a vertex of degree 2, while v_2 has degree 3 and is *not* adjacent to any vertex of degree 2, can we conclude that $G_1 \not\cong G_2$? Explain your answer.

4. Can a disconnected graph be self-complementary? Explain.

5. Consider the (unlabeled) graphs G, H_1, H_2, and H_3 below:

G:  H_1: H_2:  H_3: 

Determine which H_i are isomorphic to a subgraph of G. Explain your answer.

6. Suppose we have a collection $G_1, \ldots, G_n$ of graphs, some pairs of which are isomorphic and some pairs of which are not. Show that there is an even number of graphs that are isomorphic to an odd number of graphs. Hint: Create a graph of order n in which v_i and v_j, $i \neq j$, are adjacent if and only if G_i is isomorphic to v_j.

7. Does there exist a trio of graphs, G_1, G_2, and G_3, with exactly two of the graphs isomorphic to each other, but no other pair isomorphic to each other?

8. Prove or disprove: if G and H are two connected graphs of order n, and there exists a bijection $\phi\colon V(G) \to V(H)$ with the property that for all $u, v \in V(G)$, $d_G(u, v) = d_H(\phi(u), \phi(v))$ (that is, ϕ preserves distances), then $G \cong H$.

9. Determine all automorphisms of a path of length 5.

10. Determine all automorphisms of a cycle of length 3.