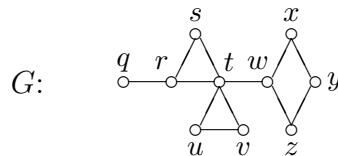


Math 483 - Spring 26

HOMEWORK 6

Due Thursday March 12.

1. Show that if a tree T has a vertex v with $\deg(v) > 2$, then T has more than two end-vertices.
2. Suppose G is a connected graph of order n , and v is a cut-vertex of G . What is the largest possible number of connected components that $G - v$ may have? Explain how to draw a graph that achieves the upper bound you give.
3. Let G be a connected graph. Prove that if a vertex v is an end-vertex of a spanning tree of G , then v is not a cut-vertex of G .
4. Let G be a connected graph with at least two vertices, and let v be a vertex of G . Show that there exists a spanning tree of G that contains all edges of G that are incident with v .
5. Prove that if G is a connected graph that has exactly two vertices that are not cut-vertices, then G is a path.
6. For the graph G below, determine the cut-vertices, bridge, and blocks of G :



7. Let G be a connected graph, v a cut-vertex of G , and let G_1 be a connected component of $G - v$. Show that the induced subgraph of G determined by $V(G_1) \cup \{v\}$ (that is, the vertices of G_1 plus the vertex v) is connected.
8. Show that if v is a cut-vertex of G , then v is not a cut-vertex of the complement \overline{G} of G .