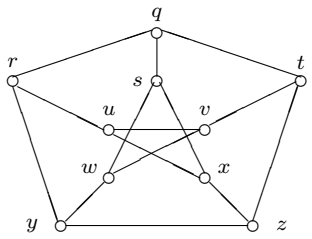
Math 483 - Spring 26

HOMEWORK 7

Due Thursday April 2.

1. Consider the Petersen Graph:



   (i) Give an example of a minimum vertex cut for the Petersen Graph.

   (ii) Give an example of a vertex cut U for the Petersen Graph that is not a minimum vertex cut, and such that no proper subset of U is a vertex cut for the Petersen Graph.

2. Give an example of a 2-connected graph that is not 3-connected.

3. Give an example of a 2-edge connected graph that is not 3-edge connected.

4. Prove that if G is k-connected, and v is a vertex of G, then $G - v$ is $(k-1)$-connected.

5. Prove that if G is ℓ-edge connected and e is an edge of G, then $G-e$ is $(\ell-1)$-edge connected.

6. Let G be a graph of order n, and let v be a vertex of G. Prove that if $\deg_G(v) = n-1$ and U is a vertex cut for G, then $v \in U$.

7. Let G be a 5-connected graph and let u, v, and w be three distinct vertices of G. Prove that G contains two cycles, C and C', that have only u and v in common, and neither of them contains w.

8. Suppose that G is a k-connected graph, and u, $v_1, \ldots, v_k$ are $k+1$ distinct vertices of G. Show that we can find path P_1 from u to v_1, P_2 from u to v_2, ..., and P_k from u to v_k such that $P_1, \ldots, P_k$ are internally disjoint.