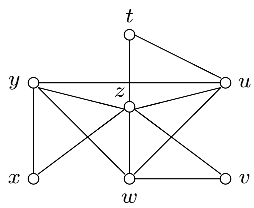
Math 483 - Spring 26
HOMEWORK 8
Due Thursday 16

1. For each of the following, give an example of a graph G satisfying the following conditions (here, $\overline{G}$ represents the complement of G):
   (i) A graph G such that both G and $\overline{G}$ have Eulerian circuits.
   (ii) A graph G that has an Eulerian circuit, but $\overline{G}$ does not.
   (iii) A graph G such that neither G nor $\overline{G}$ contain an Eulerian circuit, and both G and $\overline{G}$ do contain an Eulerian trail.
   (iv) A graph G such that neither G nor $\overline{G}$ contain an Eulerian circuit, G *does* contain an Eulerian trail, and $\overline{G}$ does not contain an Eulerian trail.

2. Show that if G is connected and has order at least 2, then there is a closed walk in G that includes every vertex at least once, and every edge exactly twice.

3. Let G be the graph below:

   Determine whether G is Hamiltonian or not; if it is Hamiltonian, explicitly write out a Hammiltonian cycle. If it is not, prove that no Hamiltonian cycle exists.

4. Recall that if G and H are graphs, then $G + H$ is the obtained by taking a copy of G, a copy of H, and then adding an edge between each vertex of G and every vertex of H.

   Suppose that G is a 3-regular graph of order 12, and H is a 4-regular graph of order 11. Is $G + H$ Eulerian?

5. Give $C(K_{2,3})$, the completion of the complete bipartite graph with parts of order 2 and 3.

6. Let G be a connected r-regular graph of even order n such that $\overline{G}$ is also connected.
   (i) Prove that either G or $\overline{G}$ are Eulerian, but not both.
   (ii) Prove that either G or $\overline{G}$ are Hamiltonian.