Math 483 - Spring 26

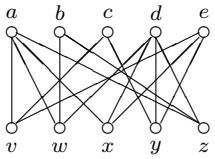
HOMEWORK 9

Due Thursday April 23

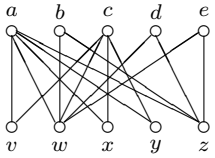
1. Give an example of a graph G such that:
   (i) G has an Eulerian circuit, but no Hamiltonian cycle (explain why it does not have a Hamiltonian cycle).
   (ii) G has a Hamiltonian cycle, but no Eulerian circuit (explain why it does not have an Eulerian circuit).
   (iii) G has a Hamiltonian cycle and an Eulerian trail, but no Eulerian circuit.
   (iv) G has neither an Eulerian circuit nor a Hamiltonian cycle, but does have an Eulerian trail.

2. Let G be a graph of order $n \geq 3$ such that for every pair of nonadjacent vertices $u \neq v$, $\deg(v) + \deg(v) \geq n - 1$. Prove that G contains a Hamiltonian path. HINT: Add a vertex x that is adjacent to every vertex of G and show that the resulting graph has a Hamiltonian cycle.

3. Let G be the bipartite graph below with partite sets $U = \{v, w, x, y, z\}$ and $W = \{a, b, c, d, e\}$. Determine if U can be matched to W:



4. Let G be the bipartite graph below with partite sets $U = \{v, w, x, y, z\}$ and $W = \{a, b, c, d, e\}$. Determine if U can be matched to W:



5. Show that a tree has at most one perfect matching.

6. Show that every connected graph of order 4 has a perfect matching, except for $K_{1,3}$.

7. Let G be a connected graph of even order. Prove that if G contains no induced subgraph that is isomorphic to $K_{1,3}$, then G has a perfect matching.

8. Show that K_4 has two perfect matchings that are disjoint (no edge is contained in both matchings).

9. Show that the Petersen graph has a perfect matching.

10. Show that the Petersen graph does not contain two perfect matchings that are disjoint. HINT: Recall that the smallest cycle in the Ptersen graph has length 5.