Math 561 - Homework 7

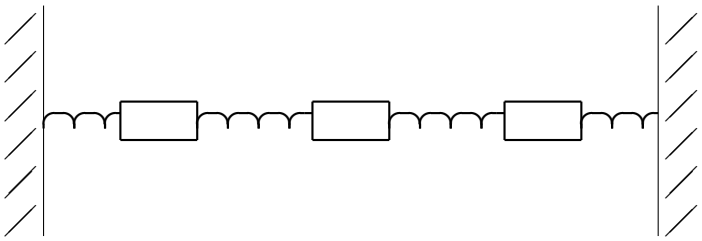
Due Wednesday November 11

1. Griffins and Dragons live in the Enchanted Forest. The number of Dragons D and of Griffins G in the forest each year is determined by their populations the previous year, according to the formulas

$$\begin{aligned} D(n+1) &= 1.5D(n) + G(n) \\ G(n+1) &= D(n). \end{aligned}$$

   If in year 1 there are 25 Dragons and no Griffins, what will the population be in year k? (Don't worry about fractional answers: mythical creatures do not have to come in whole units). In the long run, will the total population grow, shrink, or approach a nonzero equilibrium point? After a very long time, approximately what will be the ratio of Dragons to Griffins?

2. Consider three coupled blocks illustrated below; each block has mass $m = 1$ and each spring has spring constant $k = 1$. Write down the equations of motion for this coupled system, and describe the corresponding eigenvalues and eigenvectors.



3. The State University State football team gets confident after each win, and demoralized after each loss. After winning a game, it has a 90% chance of winning the next game, but after a loss it has only a 20% chance of winning the next game. In the long run, what fraction of the games can we expect the team to win?

4. The City State of Ankh-Morpork has a population of 9 million, of whom 5 million live in the Old City of Ankh, and 4 million live in the Morpork neighborhoods. Each year, only 20% of the Morporkians move to Ankh (and 80% remain in Morpork), while 40% of Ankhites move to Morpork (and 60% stay in Ankh). Citizens of Ankh-Morpork, however, must remain in Ankh-Morpork (they are not allowed to migrate outside the Twin Cities), and nobody *wants* to move *into* Ankh-Morpork. If we ignore the effects of births and deaths, what percentage of the total population will the Old City of Ankh have in the long run?

5. Three Knights of the Round Table sit down for dinner (at the Round Table, of course). They are each brought a bowl of porridge. Unhappy with their portions, each Knight takes a spoon in each hand, and simultaneously takes half the porridge from the Knight to their left and half the porridge from the Knight on their right, and places it on his own bowl (which has now been emptied by his two adjoining Knights). Describe what happens to the distribution of porridge among the bowls if this process is repeated indefinitely.

6. Repeat the previous problem with 4 Knights.