Math 666 - Homework 1
Due Wednesday February 12

1. **Exercise 8.3:3.** Let $\mathcal{C}$ be a category with small coproducts (that is, any family of objects of $\mathcal{C}$ that is indexed by a small set has a coproduct in $\mathcal{C}$), and let $U\colon \mathcal{C} \to \mathsf{Set}$ be a functor. Prove that U has a left adjoint if and only if U is representable.

2. **Exercise 8.3:5.** Show that if $A\colon \mathcal{C} \to \mathcal{D}$ and $B\colon \mathcal{D} \to \mathcal{C}$ give an equivalence of categories, then B is both a right and a left adjoint of A.

3. **Exercise 8.3:6.** Let $\mathcal{C}$ be the category with $\mathrm{Ob}(\mathcal{C}) = \mathrm{Ob}(\mathsf{Group})$, but with morphisms defined so that for any groups G and H, $\mathcal{C}(G, H) = \mathsf{Set}(|G|, |H|)$. Thus, Group is a subcategory of $\mathcal{C}$ with the same objects, but smaller morphism sets. Does the inclusion function $\mathsf{Group} \to \mathcal{C}$ have a left and/or a right adjoint?