Math 666 - Homework 4
Due Wednesday February 26

1. **Exercise 8.5:9.** For this exercise, assume the known facts that

   (i) Every subgroup of a free group is free; and

   (ii) In the free group on x and y, the subgroup generated by the commutator $x^{-1}y^{-1}xy$ and $x^{-2}y^{-1}x^2y$ is free on those two generators.

   Let F be the free group on x and y, and let $f\colon F \to F$ be the endomorphism of F determined by mapping x to $x^{-1}y^{-1}xy$, and y to $x^{-2}y^{-1}x^2y$. Let G be the direct limit of the system

   $$F \xrightarrow{f} F \xrightarrow{f} F \xrightarrow{f} F \xrightarrow{f} \cdots$$

   Show that G is nontrivial, and every finitely generated subgroup of G is free, but that G equals its own commutator subgroup; that is, that the abelianization of G is the trivial group.

   Conclude that G is "locally free" but not free. (A group is "locally X" if every finitely generated subgroup of G has property X).

2. **Exercise 8.8:1**

   (i) Combine the fact that left adjoints respect colimits and right adjoints respect limits with the characterization of monomorphisms and epimorphisms in Lemma 7.8.11 (as being parts of certain pullback or pushout diagrams) to obtain results about how left and right adjoint functors behave with respect to epimorphisms and monomorphisms.

   (ii) The results you get in (i) will not state that both left and right adjoints respect both epimorphisms and monomorphisms. For each implication that was not proven by part (i) provide an example showing that the corresponding result need not hold in general.