Math 666 - Homework 7

Due Wednesday March 26

1. **Exercise 9.3:3** (partial)

   (i) Show that a free Ω-algebra is free on a unique set of generators. That is, if (F, u) is a free Ω-algebra, then the image in $|F|$ of the set map u is determined by the Ω-algebra structure of F.

   (ii) Is the analogous result true for free groups? Free monoids? Free rings?

2. **Exercise 9.3:4**

   (i) Show that every subalgebra of a free Ω-algebra F is free.

   (ii) Is the analogous statement true for monoids?

3. **Exercise 9.3:6** (partial)

   (i) Show that every functor $A\colon \mathsf{Set} \to \mathsf{Set}$ sends surjective maps to surjective maps and injective maps with nonempty domain to injective maps.

   (ii) Show that the second clause of (i) becomes false if the qualification about nonempty domains is omitted.

   (iii) Show that if A has the form UF, where $U\colon \mathcal{C} \to \mathsf{Set}$ is some functor from a category $\mathcal{C}$, and F is a left adjoint of U, then A carries maps with empty domain to injective maps.