For the Ausoni-Rognes conjecture at $n = 1, p > 3$: a strongly convergent descent spectral sequence

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June 2nd, 2015

- $n \geq 1$
- p, a prime
- E_n is the Lubin-Tate spectrum, with $\pi_*(E_n) = W(\mathbb{F}_{p^n})[\![u_1, \ldots, u_{n-1}]\!][u^{\pm 1}]$. Here:
  - $W(\mathbb{F}_{p^n})$ is the ring of Witt vectors of the field $\mathbb{F}_{p^n}$
  - the complete power series ring is in degree zero
  - $|u| = 2$
    (this is the "choice" Ausoni makes in his Inventiones paper)
- $\mathbb{G}_n = S_n \rtimes \mathrm{Gal}(\mathbb{F}_{p^n}/\mathbb{F}_p)$, the extended Morava stabilizer group
- E_n, a commutative S–algebra
- $\mathbb{G}_n$ acts on E_n by maps of commutative S–algebras.
- (The preceding two points are an application of the Goerss-Hopkins-Miller Theorem.)

- (A, a commutative S–algebra) $\mapsto$ ($K(A)$, the algebraic K–theory spectrum of A, a commutative S–algebra)
  - $\Longrightarrow K(E_n)$, a commutative S–algebra
- By the functoriality of $K(-)$, $\mathbb{G}_n$ acts on $K(E_n)$ by maps of commutative S–algebras.
- $L_{K(n)}(S^0)$, the Bousfield localization of the sphere spectrum with respect to $K(n)$, the nth Morava K–theory spectrum.
- $\mathbb{G}_n$, a profinite group (more: a compact p-adic analytic group; finite v.c.d.)
- The $K(n)$–local unit map of $K(n)$-local commut. S-algebras

$$L_{K(n)}(S^0) \to E_n$$

is a consistent profaithful $K(n)$–local profinite $\mathbb{G}_n$–Galois extension (due to Rognes, Behrens-D.).

- V_n, a finite p–local complex of type $n+1$
- $v\colon \Sigma^d V_n \to V_n$, a v_{n+1}–self-map (d, some positive integer)
- Thus, v induces a sequence

$$V_n \to \Sigma^{-d} V_n \to \Sigma^{-2d} V_n \to \cdots$$

  of maps of spectra.
- We set

$$v_{n+1}^{-1} V_n = \operatorname*{colim}_{j \geq 0} \Sigma^{-jd} V_n,$$

  the colimit of the above sequence, the mapping telescope associated to the v_{n+1}–self-map v.

Conjecture (Ausoni, Rognes)

The $\mathbb{G}_n$–Galois extension $L_{K(n)}(S^0) \to E_n$ induces a map

$$K(L_{K(n)}(S^0)) \wedge v_{n+1}^{-1} V_n \to (K(E_n))^{h\mathbb{G}_n} \wedge v_{n+1}^{-1} V_n$$

that is a weak equivalence, and associated with the target of this weak equivalence is a homotopy fixed point spectral sequence that has the form

$$E_2^{s,t} \Longrightarrow (V_n)_{t-s}((K(E_n))^{h\mathbb{G}_n})[v_{n+1}^{-1}],$$

with

$$E_2^{s,t} = H_c^s(\mathbb{G}_n; (V_n)_t(K(E_n))[v_{n+1}^{-1}]).$$

Remark

We supplement the statement of the conjecture with the following comments:

- the E_2-term

$$E_2^{s,t} = H_c^s(\mathbb{G}_n; (V_n)_t(K(E_n))[v_{n+1}^{-1}])$$

  of its spectral sequence is given by continuous cohomology;
- its object $(K(E_n))^{h\mathbb{G}_n}$ is a continuous homotopy fixed point spectrum; and
- the conjecture is actually just a piece of the family of conjectures made by Ausoni and Rognes – we only stated the part that we have been focusing on.

- For every integer t, there is an isomorphism

$$(V_n)_t(K(E_n))[v_{n+1}^{-1}] \cong \pi_t(K(E_n) \wedge v_{n+1}^{-1} V_n).$$

- $\Longrightarrow$ When the conjectured spectral sequence

$$E_2^{s,t} \Longrightarrow (V_n)_{t-s}((K(E_n))^{h\mathbb{G}_n})[v_{n+1}^{-1}],$$

exists, since it is *for* homotopy fixed points, there should also be an equivalence

$$(K(E_n))^{h\mathbb{G}_n} \wedge v_{n+1}^{-1} V_n \simeq (K(E_n) \wedge v_{n+1}^{-1} V_n)^{h\mathbb{G}_n}.$$

- Obtaining this equivalence and a homotopy fixed point spectral sequence

$$E_2^{s,t} = H_c^s(\mathbb{G}_n; \pi_t(K(E_n) \wedge v_{n+1}^{-1} V_n)) \Longrightarrow \pi_{t-s}\left((K(E_n) \wedge v_{n+1}^{-1} V_n)^{h\mathbb{G}_n}\right)$$

immediately implies the existence of the conjectured spectral sequence.

- To make progress on the conjecture, one obstacle that must be overcome is that there are no known constructions of the (continuous) homotopy fixed point spectra

$$(K(E_n))^{h\mathbb{G}_n}, \ (K(E_n) \wedge v_{n+1}^{-1} V_n)^{h\mathbb{G}_n}$$

for any n and p.

- Also, there are no known constructions of the two homotopy fixed point spectral sequences that we have referred to.

In this talk, we are reporting on progress on this conjecture for $n = 1$, $p \geq 5$, with

$$V_1 = V(1), \text{ the type 2 Smith-Toda complex } S^0/(p, v_1).$$

Thus, we have

$$E_1 = KU_p, \ p\text{–completed complex } K\text{–theory},$$

$$\mathbb{G}_1 = \mathbb{Z}_p^\times \cong \mathbb{Z}_p \times \mathbb{Z}/(p-1).$$

N.B.

Henceforth, we will use the term "descent spectral sequence" in place of "homotopy fixed point spectral sequence."

Theorem 1 (D.)

Let $p \geq 5$. Given any closed subgroup K of $\mathbb{Z}_p^\times$, there is a strongly convergent descent spectral sequence

$$E_2^{s,t} \Longrightarrow \pi_{t-s}\big(\big(K(KU_p) \wedge v_2^{-1}V(1)\big)^{hK}\big),$$

where

$$E_2^{s,t} = H_c^s(K; \pi_t(K(KU_p) \wedge V(1))[v_2^{-1}]),$$

with $E_2^{s,t} = 0$, for all $s \geq 2$ and any $t \in \mathbb{Z}$. Also, there is an equivalence of spectra

$$\big(K(KU_p) \wedge v_2^{-1}V(1)\big)^{hK} \simeq \operatorname*{colim}_{j \geq 0}(K(KU_p) \wedge \Sigma^{-jd}V(1))^{hK}.$$

Remarks about the theorem

- Each subgroup K is a profinite group.
- Both occurrences of $(-)^{hK}$ in the theorem are for the application of the (continuous) K–homotopy fixed points functor for the category of discrete K-spectra.
- The term "spectrum" means symmetric spectrum: the theorem, its proof, and the underlying theory are worked out in the setting of symmetric spectra of simplicial sets.
- Letting $K = \mathbb{Z}_p^\times$ gives some progress on the Ausoni-Rognes conjecture: for $n = 1$, $p \geq 5$, and $V_1 = V(1)$, we have obtained the desired descent spectral sequence

$$H_c^s(\mathbb{G}_n; \pi_t(K(E_n) \wedge v_{n+1}^{-1} V_n)) \Longrightarrow \pi_{t-s}\big((K(E_n) \wedge v_{n+1}^{-1} V_n)^{h\mathbb{G}_n}\big).$$

- The construction of $K(KU_p)^{h\mathbb{Z}_p^\times}$ remains open.

Theorem 2 (D.)

For $p \geq 5$, there is a canonical map of symmetric spectra

$$\eta\colon K(L_{K(1)}(S^0)) \wedge v_2^{-1}V(1) \to (K(KU_p) \wedge v_2^{-1}V(1))^{h\mathbb{Z}_p^\times}.$$

Remark

It is easy to see that if η is a weak equivalence and if

$$(K(KU_p) \wedge v_2^{-1}V(1))^{h\mathbb{Z}_p^\times} \simeq K(KU_p)^{h\mathbb{Z}_p^\times} \wedge v_2^{-1}V(1),$$

then there would be an equivalence

$$K(L_{K(1)}(S^0)) \wedge v_2^{-1}V(1) \simeq K(KU_p)^{h\mathbb{Z}_p^\times} \wedge v_2^{-1}V(1),$$

which would be close to proving part of the Ausoni-Rognes conjecture in the $n = 1$, $p \geq 5$ case.

For the next result ...

Let G be a profinite group, with $\mathcal{N} = \{N_\alpha\}_{\alpha \in \Lambda}$ an inverse system of open normal subgroups of G that satisfies

(a) the maps in the diagram $\mathcal{N}$ (indexed by the directed poset Λ) are given by inclusions (that is, $\alpha_1 \leq \alpha_2$ in Λ if and only if N_{α_2} is a subgroup of N_{α_1}), and
(b) the intersection $\bigcap_{\alpha \in \Lambda} N_\alpha$ is the trivial group $\{e\}$.

Also, let X be a G–spectrum such that the G–module $\pi_t(X)$ is a discrete G–module, for every $t \in \mathbb{Z}$.

Theorem (D.)

Let G and X be as on the previous slide. Suppose that the map

$$\lambda^s_{\pi_t(X)} \colon H^s_c(N_\alpha; \pi_t(X)) \to H^s(N_\alpha; \pi_t(X))$$

is an isomorphism for all $s \geq 0$, every integer t, and each $\alpha \in \Lambda$. If

- *there exists a natural number r, such that for all integers t and every $\alpha \in \Lambda$, $H^s_c(N_\alpha; \pi_t(X)) = 0$, for all $s > r$; or*
- *there exists some fixed integer l, such that $\pi_t(X) = 0$, for all $t > l$,*

then there is a zigzag of G–equivariant maps

$$X \xrightarrow{\simeq} \operatorname*{holim}_{\Delta} \mathrm{Sets}(G^{\bullet+1}, X_f) \xleftarrow{\simeq} X^{\mathrm{dis}}_{\mathcal{N}}$$

that are weak equivalences in Sp^Σ, with $X^{\mathrm{dis}}_{\mathcal{N}} \in \Sigma\mathrm{Sp}_G$.

For Theorem 1, we didn’t need the full power of the preceding theorem; we only needed a simpler version ...

Definition

A spectrum X is an f–*spectrum* if $\pi_*(X)$ is degreewise finite.

Theorem (D.)

Let p be any prime and let H be any finite discrete group. If X is a $(\mathbb{Z}_p \times H)$–spectrum and an f–spectrum, then there is a zigzag

$$X \xrightarrow{\simeq} X' \xleftarrow{\simeq} X_{\mathcal{N}}^{\mathrm{dis}}$$

of $(\mathbb{Z}_p \times H)$–spectra and $(\mathbb{Z}_p \times H)$–equivariant maps that are weak equivalences of symmetric spectra, and $X_{\mathcal{N}}^{\mathrm{dis}}$ is a discrete $(\mathbb{Z}_p \times H)$–spectrum.

$$X = X_{\mathcal{N}}^{\mathrm{dis}} \quad \Longrightarrow \quad X^{h(\mathbb{Z}_p \times H)} := (X_{\mathcal{N}}^{\mathrm{dis}})^{h(\mathbb{Z}_p \times H)}$$

The construction of $X_{\mathcal{N}}^{\mathrm{dis}}$ is elementary!:

$$X_{\mathcal{N}}^{\mathrm{dis}}$$

is equal to

$$\operatorname*{colim}_{m\geq 0}\ \operatorname*{holim}_{[n]\in\Delta}\Big(\underbrace{\mathrm{Sets}(\mathbb{Z}_p\times H,\cdots,\mathrm{Sets}(\mathbb{Z}_p\times H,}_{(n+1)\text{ times}}X_f\underbrace{)\cdots)}_{\substack{(n+1)\\ \text{times}}}\Big)^{(p^m\mathbb{Z}_p)\times\{e\}},$$

where each $(p^m\mathbb{Z}_p)\times\{e\}$ is an (open normal) subgroup of $\mathbb{Z}_p\times H$ and $p^m\mathbb{Z}_p$ has its usual meaning.

Now we build on these tools for the situation of filtered colimits.

Let G be any profinite group and X any G–spectrum.

Definition

If G, X, and $\mathcal{N}$ (an inverse system of open normal subgroups of G) satisfy the hypotheses of either of the last two theorems, then we say that the triple $(G, X, \mathcal{N})$ is *suitably finite*.

Definition

Let G be a profinite group with $\mathcal{N}$ a fixed inverse system of open normal subgroups of G, and let $\{X_\mu\}_\mu$ be a filtered diagram of G–spectra such that for each μ, $(G, X_\mu, \mathcal{N})$ is a suitably finite triple and X_μ is a fibrant spectrum. We refer to $(G, \{X_\mu\}_\mu, \mathcal{N})$ as a *suitably filtered triple*.

Let $(G, \{X_\mu\}_\mu, \mathcal{N})$ be a suitably filtered triple. There is a zigzag of G–equivariant maps

$$\operatorname{colim}_\mu X_\mu \xrightarrow{\simeq} \operatorname{colim}_\mu \operatorname{holim}_\Delta \mathrm{Sets}(G^{\bullet+1}, (X_\mu)_f) \xleftarrow{\simeq} \operatorname{colim}_\mu (X_\mu)^{\mathrm{dis}}_{\mathcal{N}}$$

that are weak equivalences in $\Sigma\mathrm{Sp}$. The composition

$$\operatorname{colim}_\mu \pi_t(X_\mu) \xrightarrow{\cong} \pi_t(\operatorname{colim}_\mu (X_\mu)^{\mathrm{dis}}_{\mathcal{N}}) \xrightarrow{\cong} \operatorname{colim}_\mu \pi_t((X_\mu)^{\mathrm{dis}}_{\mathcal{N}})$$

consists of two isomorphisms in the category of discrete G–modules (in particular, each of the above abelian groups is a discrete G–module).

Definition

Given a suitably filtered triple $(G, \{X_\mu\}_\mu, \mathcal{N})$, we have seen that the G–spectrum $\operatorname{colim}_\mu X_\mu$ can be identified with the discrete G–spectrum $\operatorname{colim}_\mu (X_\mu)^{\mathrm{dis}}_{\mathcal{N}}$. Thus, it is natural to define

$$(\operatorname*{colim}_\mu X_\mu)^{hG} = (\operatorname*{colim}_\mu (X_\mu)^{\mathrm{dis}}_{\mathcal{N}})^{hG}.$$

We can extend this definition to an arbitrary closed subgroup K in G: since the K–spectrum $\operatorname{colim}_\mu X_\mu$ can be regarded as the discrete K–spectrum $\operatorname{colim}_\mu (X_\mu)^{\mathrm{dis}}_{\mathcal{N}}$, we define

$$(\operatorname*{colim}_\mu X_\mu)^{hK} = (\operatorname*{colim}_\mu (X_\mu)^{\mathrm{dis}}_{\mathcal{N}})^{hK}.$$

We say that a profinite group G has *finite virtual cohomological dimension* ("finite v.c.d.") if G contains an open subgroup that has finite c.d.

Theorem (D.)

Let G be a profinite group with finite v.c.d. If $(G, \{X_\mu\}_\mu, \mathcal{N})$ is a suitably filtered triple and K is a closed subgroup of G, then there is a conditionally convergent descent spectral sequence $E_r^{*,*}(K)$ that has the form

$$E_2^{s,t}(K) = H_c^s(K; \pi_t(\operatorname{colim}_\mu X_\mu)) \Longrightarrow \pi_{t-s}\big((\operatorname{colim}_\mu X_\mu)^{hK}\big).$$

An almost complete sketch of the proof of this theorem ...

Let U be an open subgroup of G that has finite c.d. Then $U \cap K$ is an open subgroup of K, and since U has finite c.d. and $U \cap K$ is closed in U, there exists some r such that for any discrete $(U \cap K)$–module M,

$$H_c^s(U \cap K; M) \cong H_c^s(U; \mathrm{Coind}_{U \cap K}^U(M)) = 0, \quad \text{whenever } s > r,$$

by Shapiro's Lemma. **This shows that K has finite v.c.d.**

Then, as a special case of a result due to [Behrens-D., D.], we obtain the conditionally convergent spectral sequence

$$E_2^{s,t} = H_c^s(K; \pi_t(\operatorname*{colim}_{\mu}(X_\mu)_{\mathcal{N}}^{\mathrm{dis}})) \Longrightarrow \pi_{t-s}\Big(\big(\operatorname*{colim}_{\mu}(X_\mu)_{\mathcal{N}}^{\mathrm{dis}}\big)^{hK}\Big),$$

and **this is the desired spectral sequence.**

A little more useful detail ...

Since K has finite v.c.d.,

$$\big(\operatorname*{colim}_{\mu}(X_\mu)^{\mathrm{dis}}_{\mathcal{N}}\big)^{hK} \simeq \operatorname*{holim}_{\Delta} \Gamma^\bullet_K \operatorname*{colim}_{\mu}(X_\mu)^{\mathrm{dis}}_{\mathcal{N}},$$

and for each $m \geq 0$, the m-cosimplices of the cosimplicial spectrum $\Gamma^\bullet_K \operatorname{colim}_\mu (X_\mu)^{\mathrm{dis}}_{\mathcal{N}}$ satisfy the isomorphism

$$\big(\Gamma^\bullet_K \operatorname*{colim}_{\mu}(X_\mu)^{\mathrm{dis}}_{\mathcal{N}}\big)^m \cong \operatorname*{colim}_{V \lhd_o K^m} \textstyle\prod_{K^m/V} \displaystyle\operatorname*{colim}_{\mu}(X_\mu)^{\mathrm{dis}}_{\mathcal{N}},$$

where K^m is the m-fold Cartesian product of K (K^0 is the trivial group $\{e\}$, equipped with the discrete topology).

Our spectral sequence is the homotopy spectral sequence for the spectrum

$$\operatorname*{holim}_{\Delta} \Gamma^\bullet_K \operatorname*{colim}_{\mu} (X_\mu)^{\mathrm{dis}}_{\mathcal{N}}.$$

Based on [Behrens-D., D.], **one might expect us to instead form** the homotopy spectral sequence for

$$\operatorname*{holim}_{\Delta} \Gamma^\bullet_K \big(\operatorname*{colim}_{\mu} (X_\mu)^{\mathrm{dis}}_{\mathcal{N}}\big)_{fK}.$$

But since each $(X_\mu)^{\mathrm{dis}}_{\mathcal{N}}$ is a fibrant spectrum, $\mathrm{colim}_\mu (X_\mu)^{\mathrm{dis}}_{\mathcal{N}}$ is already a fibrant spectrum, so that **we do not need to apply $(-)_{fK}$ to it (so that we are taking the homotopy limit of a cosimplicial fibrant spectrum).**

This completes our sketch-proof.

Theorem (D.)

Let G be a profinite group with finite v.c.d., let $(G, \{X_\mu\}_\mu, \mathcal{N})$ be a suitably filtered triple such that $\{\mu\}_\mu$ is a directed poset, and let K be a closed subgroup of G. If there exists a nonnegative integer r such that for all $t \in \mathbb{Z}$ and each μ, $H^s_c(K; \pi_t(X_\mu)) = 0$ whenever $s > r$, then descent spectral sequence $E_r^{*,*}(K)$ is strongly convergent and there is an equivalence of spectra

$$(\operatorname*{colim}_\mu X_\mu)^{hK} \simeq \operatorname*{colim}_\mu (X_\mu)^{hK}.$$

We give the complete proof of this result ...

For all $t \in \mathbb{Z}$, when $s > r$, we have

$$E_2^{s,t}(K) = H_c^s(K; \pi_t(\operatorname{colim}_{\mu} X_\mu)) \cong \operatorname{colim}_{\mu} H_c^s(K; \pi_t(X_\mu)) = 0,$$

so that the spectral sequence is strongly convergent, by [Thomason's Lemma 5.48].

If V is an open normal subgroup of K^m, where $m \geq 0$, then K^m/V is finite, and hence, an earlier isomorphism implies that

$$\begin{aligned}\left(\Gamma_K^\bullet \operatorname*{colim}_\mu (X_\mu)^{\mathrm{dis}}_{\mathcal{N}}\right)^m &\cong \operatorname*{colim}_\mu \operatorname*{colim}_{V \lhd_o K^m} \textstyle\prod_{K^m/V} (X_\mu)^{\mathrm{dis}}_{\mathcal{N}} \\ &\cong \operatorname*{colim}_\mu \left(\Gamma_K^\bullet (X_\mu)^{\mathrm{dis}}_{\mathcal{N}}\right)^m,\end{aligned}$$

so that there is an isomorphism

$$\Gamma_K^\bullet \operatorname*{colim}_\mu (X_\mu)^{\mathrm{dis}}_{\mathcal{N}} \cong \operatorname*{colim}_\mu \Gamma_K^\bullet (X_\mu)^{\mathrm{dis}}_{\mathcal{N}}$$

of cosimplicial spectra.

Therefore, we have

$$\left(\operatorname*{colim}_{\mu}(X_\mu)^{\mathrm{dis}}_{\mathcal{N}}\right)^{hK} \simeq \operatorname*{holim}_{\Delta} \Gamma^\bullet_K \operatorname*{colim}_{\mu}(X_\mu)^{\mathrm{dis}}_{\mathcal{N}} \cong \operatorname*{holim}_{\Delta} \operatorname*{colim}_{\mu} \Gamma^\bullet_K (X_\mu)^{\mathrm{dis}}_{\mathcal{N}},$$

which gives

$$\begin{aligned}(\operatorname*{colim}_{\mu} X_\mu)^{hK} \simeq \operatorname*{holim}_{\Delta} \operatorname*{colim}_{\mu} \Gamma^\bullet_K (X_\mu)^{\mathrm{dis}}_{\mathcal{N}} \longleftarrow & \operatorname*{colim}_{\mu} \operatorname*{holim}_{\Delta} \Gamma^\bullet_K (X_\mu)^{\mathrm{dis}}_{\mathcal{N}} \\ & \simeq \operatorname*{colim}_{\mu} \left((X_\mu)^{\mathrm{dis}}_{\mathcal{N}}\right)^{hK} \\ & = \operatorname*{colim}_{\mu} (X_\mu)^{hK},\end{aligned}$$

and **the canonical colim/holim exchange map above is a weak equivalence if there exists a nonnegative integer r such that** for every t and all μ,

$$H^s\left[\pi_t\left(\Gamma^*_K (X_\mu)^{\mathrm{dis}}_{\mathcal{N}}\right)\right] = 0, \quad \text{when } s > r.$$

The proof is completed by noting that there are isomorphisms

$$\begin{aligned} H^s\left[\pi_t\left(\Gamma_K^*(X_\mu)_{\mathcal{N}}^{\mathrm{dis}}\right)\right] &\cong H_c^s(K; \pi_t((X_\mu)_{\mathcal{N}}^{\mathrm{dis}})) \\ &\cong H_c^s(K; \pi_t(X_\mu)), \end{aligned}$$

for all $s \geq 0$.

Ausoni showed that $K(ku_p) \wedge V(1)$ is an f–spectrum.

$$\Longrightarrow$$

$K(KU_p) \wedge V(1)$ is an f–spectrum.

Then our tools give

$$K(KU_p) \wedge v_2^{-1} V(1) = \operatorname*{colim}_{j \geq 0} \left(\left(K(KU_p) \wedge \Sigma^{-jd} V(1) \right)_f \right)_{\mathcal{N}}^{\mathrm{dis}} \in \Sigma \mathrm{Sp}_{\mathbb{Z}_p^\times}$$

and

$$\left(K(KU_p) \wedge v_2^{-1} V(1) \right)^{hK} = \left(\operatorname*{colim}_{j \geq 0} \left(\left(K(KU_p) \wedge \Sigma^{-jd} V(1) \right)_f \right)_{\mathcal{N}}^{\mathrm{dis}} \right)^{hK}.$$

We have not constructed $(K(KU_p))^{h\mathbb{Z}_p^\times}$ for any p. Nevertheless, we have a "draft result" related to this conjectural object ...

A Recollection

If G is any profinite group and X is a (naive) G–spectrum, then G can be regarded as a discrete group and one can always form the "discrete homotopy fixed point spectrum"

$$X^{\widetilde{hG}} = \mathrm{Map}_G(EG_+, X).$$

"Draft theorem"

When $p \geq 5$, there is an equivalence of spectra

$$(K(KU_p) \wedge v_2^{-1}V(1))^{h\mathbb{Z}_p^\times} \simeq (K(KU_p))^{\widetilde{h\mathbb{Z}_p^\times}} \wedge v_2^{-1}V(1).$$

Does this work shed any light on the
Ausoni-Rognes Conjecture for higher n?

- For any n and p: there exists $K \lhd_c \mathbb{G}_n$, with $\mathbb{G}_n/K \cong \mathbb{Z}_p$.
- I believe it is reasonable to think that there exists an equivalence

$$(K(E_n) \wedge v_{n+1}^{-1} V_n)^{h\mathbb{G}_n} \simeq \left((K(E_n) \wedge v_{n+1}^{-1} V_n)^{hK}\right)^{h\mathbb{Z}_p}.$$

Also, if

- $K(E_n) \wedge V_n$ can be shown to be an f-spectrum, and
- $(K(E_n) \wedge V_n)^{hK}$ can be constructed as an f-spectrum,

then I believe that the tools and techniques of this work will yield a construction of

$$\left((K(E_n) \wedge v_{n+1}^{-1} V_n)^{hK}\right)^{h\mathbb{Z}_p}.$$