

Let  $H$  be a nontrivial proper closed subgroup of  $G$ . If  $H$  is open in  $G$ , then, as sorta implicitly explained below, the fibrant model in my paper can be obtained from Jardine's Godement resolution approach.

But suppose that  $H$  is closed, and not open, in  $G$ . I'm not certain of the following assertion, but I think that, in this case, Jardine's Godement approach does not give the fibrant model described in my paper. Let  $T_H$  be the category of discrete  $H$ -sets. Again, I'm not certain of the following, but I think that the functor

$$\text{Map}_c(G, -): \mathbf{Sets} \rightarrow T_H$$

would have to be a right adjoint for Jardine's approach to yield the paper's fibrant model. But this functor is usually not going to be a right adjoint.

Suppose that  $\text{Map}_c(G, -)$ , as defined above, is a right adjoint. Then it commutes with all limits. Since we will be working with limits, it is easier to work with the functor  $\text{Map}_c(G, -)$  by identifying it with the functor

$$\text{colim}_N \prod_{G/N} (-): \mathbf{Sets} \rightarrow T_H,$$

where the colimit is taken over all the open normal subgroups of  $G$ . Let's use  $\lim$  for limits in  $\mathbf{Sets}$  and  $\lim^H$  for limits in  $T_H$ . Then, given a diagram  $\{X_i\}_i$  of sets, we require that

$$\text{colim}_N \prod_{G/N} \lim_i X_i \cong \lim_i^H \text{colim}_N \prod_{G/N} X_i.$$

Thus, we want to show that

$$(1) \quad \text{colim}_N \prod_{G/N} \lim_i X_i \cong \text{colim}_K (\lim_i \text{colim}_N \prod_{G/N} X_i)^K,$$

where  $\text{colim}_K$  indicates that the colimit is taken over all open normal subgroups  $K$  of  $H$ .

But I don't see how to verify (1) when  $H$  is closed and not open. It is possible to see why (1) fails to be true in this case, by going through the proof when  $H$  is open and noting the difference with the case when  $H$  is not open.

So now suppose that  $H$  is open and let's verify (1), beginning with the right-hand side of (1):

$$\begin{aligned} \text{colim}_K (\lim_i \text{colim}_N \prod_{G/N} X_i)^K &\cong \text{colim}_K \lim_i \text{Map}_c(G, X_i)^K \\ &\cong \text{colim}_K \lim_i \prod_{G/K} X_i \\ &\cong \text{colim}_K \prod_{G/K} \lim_i X_i \\ &\cong \text{colim}_{U <_o H} \prod_{G/U} \lim_i X_i \\ &\cong \text{colim}_{V <_o G} \prod_{G/V} \lim_i X_i \\ &\cong \text{colim}_N \prod_{G/N} \lim_i X_i, \end{aligned}$$

where the second isomorphism uses that  $G/K$  is a finite product (since  $K$  is open in  $G$ , which won't necessarily be true when  $H$  is closed and not open in  $G$ ) and the last three isomorphisms are by cofinality - the fifth isomorphism (the second application of cofinality) will not be true when  $H$  is not open in  $G$ .

So (1) need not hold when  $H$  is closed, so the requisite functor need not be a right adjoint.