# FILTERING-OUT AREA-POINT ERRORS FROM RADAR-RAINGAUGE VERIFICATION SAMPLES

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## Abstract

Large differences between area-averaged rainfall and raingauge measurements pose major difficulties in conclusive verification of radar rainfall (RR) products. We develop a procedure to account for these differences using a conditional distribution transformation (CDT) method. Its purpose is to filter-out the raingauge representativeness errors from a radar-raingauge sample and to provide the estimate of the distribution of the radar products and the corresponding true area-averaged precipitation. Retrieval of this "true verification distribution" allows application of the distribution-oriented verification methodologies to evaluate RR products in a systematic and comprehensive manner. We present the CDT method and empirical tests of its validity. The tests demonstrate that the method performs with satisfactory accuracy, and thus, can considerably improve on the currently used RR verification practices.

**Key Words: Radar rainfall, uncertainties, ground reference, error filtering, nonparametric**

## Introduction

The performance of all hydrological applications is highly dependent on the quality of rainfall estimates. Also, modern forecasting models require explicit definition of the input errors. However, quantitative characterization of all the systematic and random discrepancies between radar rainfall (RR) estimates and the corresponding truth is still problematic. One cause of this situation is the necessity of using raingauge point measurements for evaluation of the area-averaged RR estimates (Kitchen and Blackall 1992, Brandes et al. 1999, Ciach and Krajewski 1999, Habib and Krajewski 2002). To account for the area-point errors in the radar versus raingauge comparisons, Ciach and Krajewski (1999) proposed an error variance separation (EVS) method. This approach was later used in several studies (Anagnostou et al. 1999, Young et al. 2000, Habib and Krajewski 2002, Siriluk et al. 2003). However, an empirical test by Ciach et al.(2003) showed that the major assumption of the EVS method, i.e. negligible covariance between the radar and raingauge errors, is not fulfilled. Thus, the results of applying the EVS method to RR verification can be inaccurate. Besides, the EVS is limited to estimating the RR error variance only, which is not enough to describe all important aspects of the uncertainties in various RR products (Ciach et al. 2000).

For the complete statistical characterization of RR uncertainties at a given spatiotemporal scale and a given distance from the radar, it is sufficient to determine the bivariate distribution of RR estimates and the corresponding true values of rainfall averaged over the same spatiotemporal domain, called the "true verification distribution." Such distributions are rarely available since usually only data from single raingauges can be used for the verification of RR products. The approach presented here uses such typical data, together with additional information about spatial rainfall variability, to retrieve the true verification distributions. In other words, it can filter out the raingauge representativeness errors from the typical RR verification samples. We investigate a conditional distribution transformation (CDT) concept to achieve this goal. The nonparametric point-area transformation scheme used in the CDT method was first proposed by Journel and Huijbregts (1978). It was further developed by Morrissey (1991) and applied to the verification of satellite rainfall products (Morrissey and Greene 1993). Based on a simulation experiment, they evaluated the performance of their procedure and reported satisfactory results. In this paper, we summarize briefly the results of an empirical test of the CDT scheme applied to RR products. More detailed description of the method, its tests and implementation is presented in Habib et al. (2003).

## Point-area distribution transformation method

Our implementation of the point-area distribution transformation scheme follows in principle the methodology presented in Morrissey (1991). Assume stochastic homogeneity of rainfall within a spatial domain of area $A$. Let $R_p$ represent point rainfall with mean $E\{R_p\}$ and variance $Var\{R_p\}$, and $R_a$ represent the rainfall averaged over an area $A$ with mean $E\{R_a\}$ and variance $Var\{R_a\}$. The means of the two corresponding processes are equal and the variances can be related based on the spatial correlation, $\rho(x,y)$, of the rain-field:

$$Var\{R_a\} = \frac{Var\{R_p\}}{A^2} \int_A \int_A \rho(x,y) dx^2 dy^2 \, . \qquad (1)$$

Given the probability distribution of $R_p$ and the estimate of its spatial correlation function, we want to estimate the distribution of $R_a$ . This problem cannot be solved without additional assumptions. A crude solution can be based on parametric distributions governed by two parameters only. However, to avoid such assumptions, we apply a nonparametric transformation method proposed by Journel and Huijbregts (1978).

The probability distribution of raingauge measurements $R_p$ can be represented using a transformation that expresses $R_p$ as a function of the standard normal random variable $R_p=\phi_{Rp}(U)$. This function is approximated using the expansion:

$$\phi_{R_p}(u) \approx \sum_{i=0}^{n} \frac{\psi_i}{i!} H_i(u) \, , \qquad (2)$$

where $H_i(\cdot)$ are Hermite polynomials of the order $i$. The expansion coefficients, $\psi_i$, are fitted to the $R_p$ distribution using a procedure described in Journel and Huijbregts (1978). They are related to the mean and variance of the point rainfall:

$$\psi_0 = \mu \, , \qquad (3a)$$

$$\sum_{i=1}^{n} \frac{\psi_i^2}{i!} = Var\{R_p\} \, . \qquad (3b)$$

The main assumption of the point-area transformation scheme proposed by Journel and Huijbregts (1978) is that the function $\phi_{Ra}$ that represents the areal rainfall as a function of the standard normal random variable, has the same Hermite expansion as $\phi_{Rp}$, but its coefficients are modified by a single correction factor, $a$, in the following way:

$$\phi_{R_a}(u) \approx \sum_{i=0}^{n} \frac{\psi_i a^i}{i!} H_i(u) \, , \qquad (4)$$

where the coefficients $\psi_i$ are the same as in (2). The transformation imposed by (4) preserves the distribution mean since $a^0=1$. The correction factor, $a$, can be fitted to adjust the variance of $R_a$ that is known based on (1).

The point-area transformation procedure that we summarized above is general and can be applied either to the whole data sample, or to its sub-samples conditioned in any specific way. Since our focus is on verification of RR products, the distributions and their transformations have to be conditioned on the RR values, $R_r$. The scheme of this conditional distribution transformation (CDT) can be outlined as follows. First, the raingauge rainfall values in the data sample are grouped into sub-samples that are conditioned on a number of ranges of the RR values, $(R_p|R_r=r)$, each range centered on a RR value, $r$. The number of the sub-samples depends on the sample size. Then, the correlation functions of the point rainfall conditioned on the radar estimate value, $(\rho|R_r=r)$, are estimated. This enables the estimation of the conditional variances of areal rainfall, $Var\{R_a|R_r=r\}$. For each sub-sample, the Hermite expansion coefficients and the correction factors are estimated. Finally, these CDT functions are used to provide the desired estimates of the conditional distributions of the true area-averaged rainfall $f(R_a|R_r=r)$ based on (4).

## Data sample

The ideal data sample to empirically test the CDT method described above should be based on accurate measurements of areal rainfall averaged over the radar grids containing the raingauges. Fairly accurate approximations of the areal rainfall can be only obtained from super-dense networks of raingauges that cover uniformly a certain spatial domain. The data-set used in this study was created from the same data as those used in Ciach et al. (2003).

The raingauge data were collected from the Micronet network developed by the USDA Agricultural Research Service (ARS) that covers the Little Washita River watershed in Oklahoma. We selected three rectangular sub-domains of about $19\times18\text{-km}^2$ each within the watershed (Figure 1).

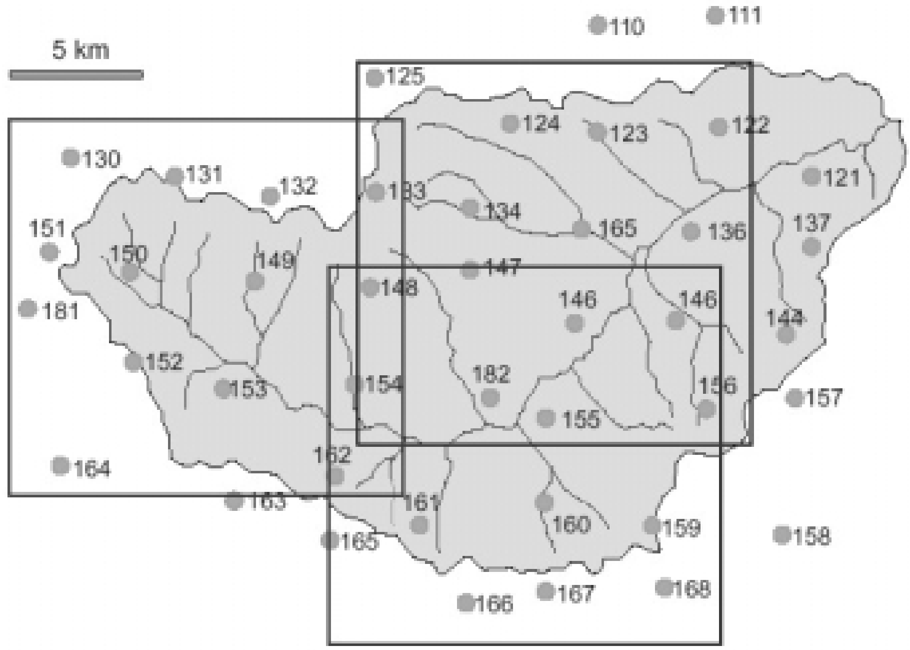


**Fig.1. A layout of the Little Washita Micronet with the three rectangular areas of about 19 km by 18 km that are used for testing the CDT method.**

Each sub-domain is covered with 15-16 stations that provide satisfactory approximations of the area-averaged rainfall (Ciach et al. 2003). Data of two warm seasons (April-September) of 1998 and 1999 were used in this study. A total rainfall accumulation of about 700 mm was collected over the watershed during this period.

The raingauge data were supplemented with RR estimates averaged over the three sub-domains to test the CDT scheme. The radar data are based on composite radar reflectivity maps produced by the NASA's Global Hydrology and Climate Center (GHRC). The maps are created from the "NOWrad© WSI's mosaic radar" products that are combined from individual radars of the NEXRAD operational network. The reflectivity-rainrate conversion was performed using a typical power-law Z-R relationship with the exponent set to 1.4 (standard NEXRAD) and the multiplier adjusted so that the sample means of the radar and raingauge areal rainfall estimates are equal.

## Test of the CDT method

To evaluate the performance of the CDT method, we used a data sample of point rainfall, areal rainfall and the corresponding RR described in the previous section. With only 12 months of observations, the sample size is relatively small. Therefore, in this study, we considered only one time scale of 15 minutes. Note also that the selected rectangular sub-domains are considerably larger than typical radar resolutions. The single raingauge representativeness errors are large in such a case, which makes the test of the raingauge error filtering method especially demanding.

We stratified the whole sample into four sub-samples according to four intervals of the 15-minute RR values, $R_r$. Due to small sample size we could consider only such a small number of the RR intervals. For each of those radar-conditioned sub-samples separately, we carried out the following test procedure:

(1) Construct the sample of concurrent point and areal rainfall. The $R_a$ values are obtained by averaging the raingauge observations within the test area, whereas the $R_p$ values come from the individual gauges.

(2) Estimate the sample variances of the point and areal rainfall values.

(3) Use the point rainfall distribution and information about $Var\{R_p\}$ and $Var\{R_a\}$ as inputs to the distribution transformation scheme described before to retrieve the conditional areal rainfall distribution.

(4) Compare the retrieved areal rainfall distribution against the observed one.

The conditional quantile-quantile plots resulting from these tests are shown in Figure 2.

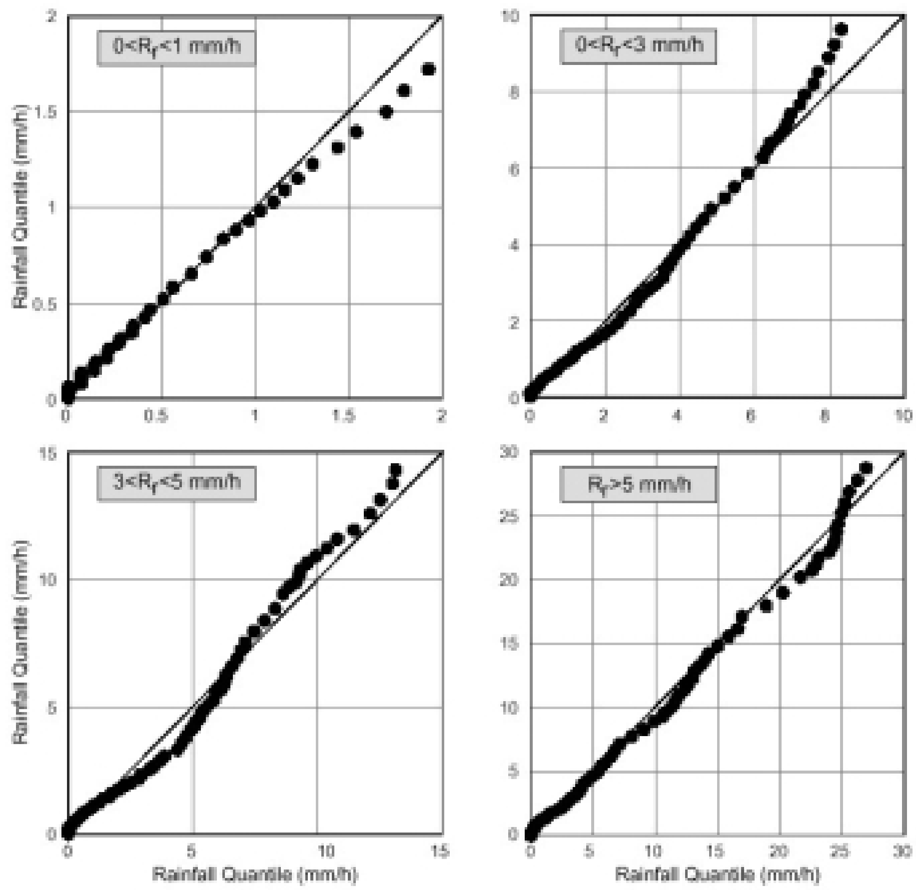


**Fig.2. A quantile-quantile plot of the cumulative rainfall distributions for the observed and the CDT-based areal rainfall. The sample is stratified into four ranges of RR values.**

The plots indicate that, overall, the transformation-based conditional distributions are in a good agreement with the observed conditional distributions of areal rainfall. Lower agreement in the region of larger rainfall values is most likely due to the sampling errors caused by small sizes of the sub-samples. Nevertheless, the test confirms the ability of the CDT method to retrieve the distribution of the areal rainfall conditioned on the corresponding radar estimates.

## Conclusions

We presented our first results on the development of a nonparametric conditional distribution transformation (CDT) method for estimation of the uncertainties in the area-averaged RR products based on their comparisons with single raingauge data. The purpose of the CDT method is filtering out the raingauge representativeness errors from the radar-gauge verification samples. We described the transformation scheme and tested its performance using large data sample from a high-density raingauge network covering Little Washita watershed in Oklahoma. The test performed in this study confirmed that the discussed point-area distribution transformation scheme was able to retrieve the conditional areal rainfall distributions from the point measurements with satisfactory accuracy.

To apply the CDT method successfully, sufficiently accurate information on the small-scale spatial rainfall correlation structure conditioned on the radar estimates is required. Our recent study on the correlation in rain-fields for the distances up to a few kilometers (Krajewski et al. 2003) suggests that larger data samples are needed to develop empirically substantiated models of the small-scale rainfall variability in different precipitation regimes. We will continue more extensive investigations of this problem in our future studies.

Another interesting issue that should be investigated in the future is the effect of spatial and temporal scales on the error distributions in RR estimates. Such analyses could also allow retrieval of the information on the spatiotemporal dependencies in the error processes. This information is essential for creating complete and realistic models of the uncertainties in RR products that could improve the performance of the ensemble forecasts, data assimilation schemes and hypothesis testing in hydrology. It is also important for the current hydrological applications of RR products that are highly dependent on their resolution.

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