Validation of the Zero-Covariance Assumption in the Error Variance Separation Method of Radar-Raingauge Comparisons

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ABSTRACT: Empirical test of zero-covariance assumption in the error variance separation method is presented. The method had been proposed to filter out ground reference errors in radar rainfall verifications. This investigation uses a large data sample of two 6-month periods from the Little Washita watershed in Oklahoma. The test results are provided with bootstrap error bounds that confirm their statistical significance. They show that, for this testing setup, the zero-covariance assumption in its previously postulated rigorous formulation is not fulfilled.

1 INTRODUCTION

Reliable, comprehensive and conclusive verification of radar rainfall products is a crucial but still unresolved problem. The rather confusing outcomes of different algorithm intercomparison efforts demonstrate that there is no agreement in our community about rigorous quantitative methods that should be applied to make the optimal choices. In such situation, qualitative guesses and/or personal preferences can negatively affect many important decisions. Several deficiencies perpetuate in current approaches to verification of the outcomes of different radar rainfall estimation procedures. Firstly, these assessments are incomplete because they base on one or two indices, ignoring other important performance measures. This is in contrast to the comprehensive methods used in weather forecasting (Murphy 1997). Secondly, the performance measures are computed from verification samples without their confidence bounds. As a result, the evaluation projects often use small data samples that are insufficient to demonstrate differences between the products. Finally, much confusion arises from neglecting the uncertainties in the data used as ground reference. The known fact that the errors of single gauges can predominate most radar-raingauge comparisons has still little effect on the common practices.

However, developing a comprehensive and reliable methodology for verification of radar rainfall products is a very complex task. It requires solving many specific questions, as well as careful and rigorous tests of every component constituting the verification process. Progress in this area must be substantiated using experimental setups of special precision. One example of such a system is the PicoNet (see http://www.evac.ou.edu/piconet) that has been recently deployed by the Oklahoma University over the Oklahoma City Airport field. It consists of 25 carefully maintained double-raingauge stations covering a 3x3 km area, and provides area-rainfall measurements with an exceptional accuracy. Another example of a high-precision network design, created in Great Britain and covering an area of about 10x10 km, is described by Moore *et al.* (2000). In larger scales, dense networks of some experimental watersheds can also be used to investigate the assumptions in the verification procedures, provided their rainfall data are of good quality. Thorough tests not always fully confirm validity of the methods that previously seemed natural and intuitively correct. In this study, we present one example of such an investigation of a technique used for separation of raingauge errors from the variance of radar-raingauge differences.

Using uncertain ground reference (GR) for evaluation of radar rainfall products is a technical necessity. The raingauge data are subject to instrumental errors and suffer from the inherent representativeness errors (or area-point sampling differences). The former can be corrected (Nešpor and Sevruk 1999), whereas the later can be reduced only by increasing the density of the networks. Since most of the verification tasks have to be based on typical relatively sparse raingauge systems, it is necessary to recognize their errors and to account for them. Ciach and Krajewski (1997) proposed a seemingly natural error variance separation (EVS) method that can be applied to verification of remote sensing rainfall estimates based on uncertain raingauge reference. Later, they implemented it on radar rainfall estimates over a broad range of spatiotemporal scales (Ciach and Krajewski 1999). The EVS method has also been applied by Anagnostou *et al.* (1999), Krajewski *et al.* (2000), Young *et al.* (2000), and

Habib and Krajewski (2001). The EVS method is simply a partitioning of the variance of the GR and radar rainfall differences into its two basic components: the GR error variance and the radar error variance. Its central assumption is that the covariance of these two errors is negligible (we call this assumption the "zero-covariance hypothesis"). In their implementation study, Ciach and Krajewski (1999) discussed several theoretical arguments that make the hypothesis plausible. Here, this question is investigated empirically using an accurate approximation of the area-rainfall based on a dense raingauge network.

2 PROBLEM STATEMENT

Detailed description of the EVS method is presented in Ciach and Krajewski (1999). Suppose that quasi-point raingauge measurements are applied as GR to verify radar rainfall products. If R_r is a random variable representing radar rainfall, R_g stands for the concurrent and collocated raingauge accumulations, and R_t is the true rainfall over the grid area of the radar product, then the variance of the radar-raingauge difference can be decomposed in the following way:

$$\mathbf{V}\{D_{rg}\} = \mathbf{V}\{E_r\} - 2\,\mathbf{Cov}\{E_r, E_g\} + \mathbf{V}\{E_g\}\,, \tag{1}$$

where $D_{rg}=R_r-R_g$ is the difference of the radar and raingauge rainfall, $E_r=R_r-R_t$ is the error of the radar rainfall, $E_g=R_g-R_t$ is the raingauge error, and $\mathbf{V}\{\cdot\}$ and $\mathbf{Cov}\{\cdot,\cdot\}$ are the variance and covariance, respectively. We assume a fixed averaging interval for all the variables stated above. The left-hand term in Eq. (1) can be estimated directly from a sample of radar and raingauge data. If the inter-gauge correlations provide enough information to retrieve the rain-field spatial correlation structure, the variance of the raingauge errors E_g can be estimated using spatial statistics. The zero-covariance hypothesis states that the covariance term in Eq. (1) is small so that the error variance of the radar rainfall can be estimated using the following formula:

$$\mathbf{V}\{E_r\} \stackrel{?}{=} \mathbf{V}\{D_{rg}\} - \mathbf{V}\{E_g\}\,, \tag{2}$$

with accuracy sufficient for its practical applications to verification and quality assessment of radar products. For this to be true, it is enough that:

$$\left|\,2\,\mathbf{Cov}\{E_r, E_g\}\right| \ll \mathbf{V}\{E_r\}\,. \tag{3}$$

Proportion of the two sides of this inequality determines how much the possible non-zero values of the covariance distort the retrieval of the radar errors based on Eq. (2).

3 DATA SAMPLE

This study is based on a large data sample covering two six-month periods (April-September) of the years 1998 and 1999. The raingauge data come from a dense network covering the Little Washita River watershed located in Oklahoma. It is an experimental basin (about 1200 km^2) equipped with 43 Micronet multi-sensor stations deployed and operated by the USDA ARS (Starjs *et al.* 1996). The average distance between the neighboring stations is about 5 km and precipitation data with 5-minute temporal resolution are collected from each station. An outline of the network is shown in Figure 1, together with the Micronet station identification numbers (we refer to them later) and the rectangular test area of about 800 km^2 that we selected for this study.

The radar rainfall data that we use here are based on radar reflectivity maps obtained from the NASA Global Hydrology Research Center. They are nation-wide mosaic composites computed from the NEXRAD Level III data of the WSR-88D radars. These products are created by the Weather Service International (WSI), and have spatial resolution of 2x2 km and temporal resolution of 15 minutes. The reflectivities are delivered in 16 levels every 5 dBZ starting from 0 dBZ (WSI 1995). Since radar rainfall estimates that we investigate are averaged over the test area, these quantisation errors average out to some degree. To convert the radar reflectivities into the radar rain-rates, we used a typical power-law Z-R relationship. We assumed its exponent value as 1.4 and adjusted its multiplier so that the sample averages of R_r and R_t are equal.

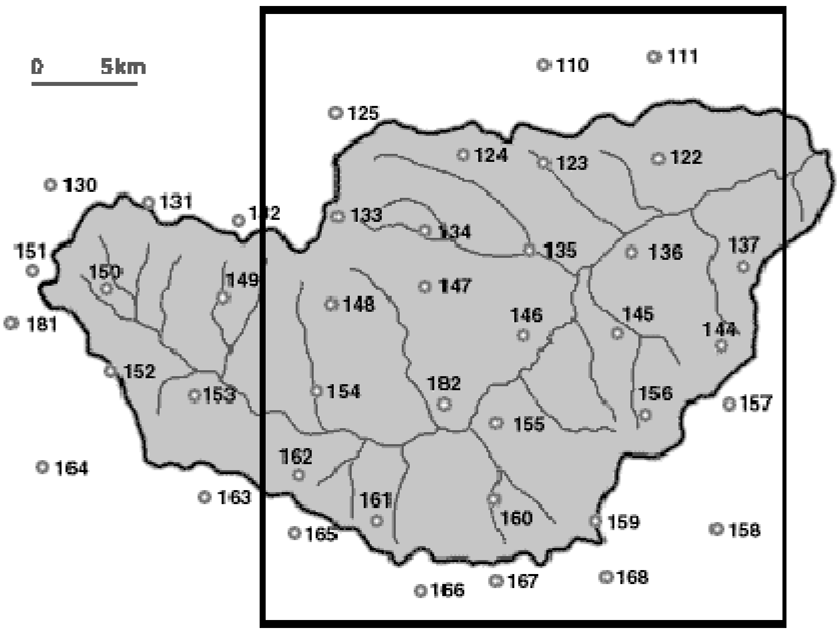

Figure 1. The Little Washita River watershed with the USDA ARS Micronet stations. The superimposed rectangle indicates the test area selected for this investigation.

4 INVESTIGATION METHOD

We answer the question stated in Section 2 in a direct way, through estimation of the terms in Eq. (3) based on the data sample described above. Next, we compare the estimates for selected locations of the point measurements and for different accumulation intervals. Good approximations of R_t are obtained as averages of the raingauge values within the test area. It has the size of about 34 km by 24 km and is covered fairly uniformly with 29 raingauges (see Figure 1). In addition to the statistics in Eq. (3), we also obtained their bootstrap distributions to assess the statistical significance of the results. We did it by random drawing with replacement from the data sample and computing the statistics in Eq. (3) for each of these bootstrap pseudo-samples. We repeated this resampling procedure 1000 times for each case to provide uncertainty bounds for the discussed results.

5 BASIC RESULTS AND DISCUSSION

In the upper panels in Figure 2, we present 2 schematic maps of sample-estimated values of the expression:

$$\frac{2\mathbf{Cov}\{E_r, E_g\}}{\mathbf{V}\{E_r\}}, \qquad (4)$$

for each of the Micronet raingauges in the test area (see Figure 1) and for 2 rainfall accumulation intervals. These proportions of two terms in Eq. (1) that we are directly concerned with, the covariance term and the radar error variance, are direct indicatiors of how much the zero-covariance hypothesis is violated. The lower panels contain the $\pm 2\sigma$ bootstrap confidence bounds of these sample-estimated ratios.

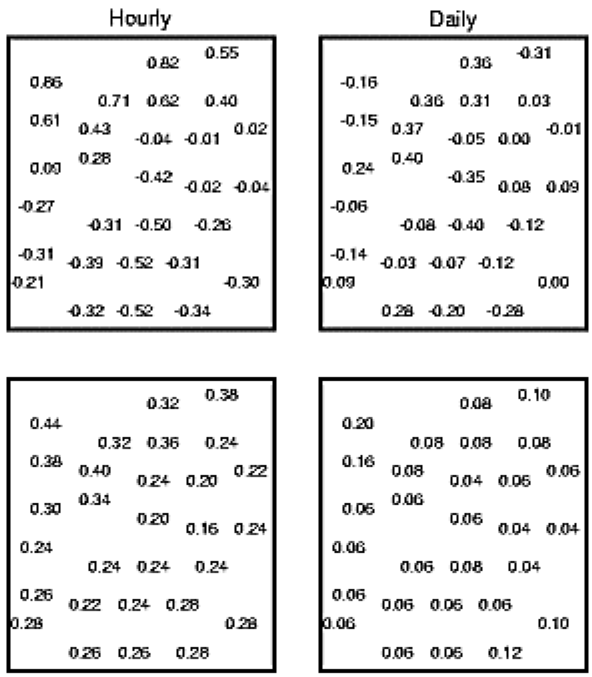

Figure 2. Upper panels: estimates of the covariance factor normalized by radar error variance.
Lower panels: 95% bootstrap confidence bounds of the above results.

Although we show above only results for two intervals, we have to conclude that the zero-covariance hypothesis, as stated formally by Eq. (3), is generally *not* fulfilled. We can see that, for a few locations of the point measurements in the test area, the magnitude of the covariance term is almost comparable with the radar error variance. The proportions in Figure 2 exhibit a fairly regular spatial behavior. They change from the prevailing positive values in the North and Northwest part of the test area to negative values over the center and in its South part. Specifically, for raingauge 146, which is close to the area center, the covariance term in Eq. (1) is negative. Its value is −42% (±20%) of the radar error variance for the hourly accumulations, and −35% (±6%) for the daily intervals (the numbers in the parentheses are the $\pm 2\sigma$ error bounds). If the EVS is applied to filter our the area-point error of gauge 146, the resulting estimates of radar error variances will be overestimated by about 40%. This level of overestimation of the errors is not excessive and could be perhaps tolerated in most practical applications.

On the other hand, for raingauge 124, the covariance term is positive and constitutes 71% (±32%) and 36% (±8%) of the radar error variance for hourly and daily intervals, respectively. For this location of the point measurements, the EVS results are strongly underestimated and, for hourly intervals, the EVS estimated radar error variance is about 3 times smaller than its true value. It might result in wrong decisions based on unjustified confidence in the overall performance of the radar rainfall products. However, for the data sample and spatial scale analyzed here, such strong underestimation occurs only for a few peripheral locations of the point measurements in the North and Northwest area of the test area. It seems that the EVS, although formally not fulfilled, can give useful results, if one avoids the gauges that are too far from the grid-center of the verified area-rainfall.

Regarding the statistical significance of these results, the lower panels in Figure 2 show that, for all the cases where the covariance departure from zero is lager than 30% of the radar error variance, zero value is outside of the

$\pm 2\sigma$ bootstrap error bounds. This indicates that our estimates of the larger departures of the covariance terms from zero are statistically significant. Thus , the data sample used here is of sufficient size to formally reject the zero-covariance hypothesis stated by Eq. (3) with high level of confidence.

6 CONCLUSIONS AND DIRECTIONS

We presented an empirical test of the zero-covariance hypothesis in the error variance separation (EVS) method of error estimation in radar rainfall. The investigation shows that this assumption is not fulfilled in our particular testing setup. However, closer examination of the effects of the non-zero error covariances on the EVS outcomes shows that they are tolerable in many practical applications. For example, for the central gauge, the EVS-estimated error variance of the radar rainfall is only about 40% higher than its true value. A necessary condition for this small effect is that gauges that are far from the grid center are avoided in radar-raingauge comparisons. Generality of this conclusion should be further investigated for other spatial resolutions, radar products and rainfall regimes. The observed regularities could then be used to correct the covariance-induced biases. Although the EVS method seems to be an intuitively straightforward way to reduce the impact of the ground reference (GR) errors, violation of its zero-covariance assumption implies using it with great caution.

Filtering out the GR uncertainties in radar rainfall verification is a question that should be pursued, if we want to evaluate the products in a meaningful way. Certainly, other methods of the error filtering are possible and should be investigated. To address these issues, we need large data samples based networks that can provide sufficiently accurate approximations of the true area-averaged rainfall. Good quality data from such systems would enable real progress in the verification methodologies of the radar rainfall products.

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