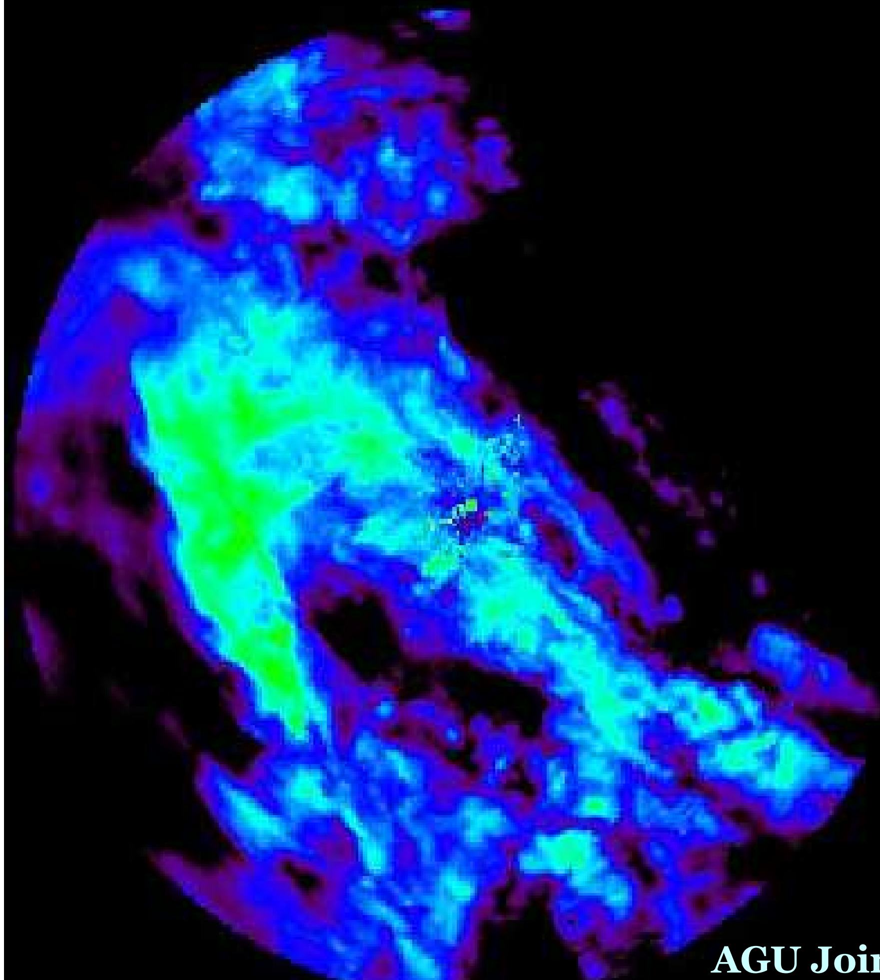
COPULA BASED SPACE-TIME RAINFALL SIMULATION



Amir Aghakouchak[1]
András Bárdossy[2]
Emad Habib[1]

[1] Department of Civil Engineering
University of Louisiana at Lafayette
[2] Department of Civil Engineering
University of Stuttgart

AGU Joint Assembly, Fort Lauderdale, USA 27-30 May 2008

Content

- **Motivation**
- **Challenges in Rainfall Simulation**
- **Methodology**
- **An Introduction to Copulas**
- **Rainfall Simulation Using:**
  - Gaussian copula
  - t-Copula
  - Clayton Copula
  - Frank Copula
  - Gumble Copula
- **Stochastical Characteristics of Simulated fields**

Motivation

Applications of rainfall Simulation in Hydrology and Meteorology:

- **Uncertainty analysis of hydrological and meteorological models due to uncertainty of input precipitation**
- **Ensemble forecasting**
- **Risk analysis and management**
- **Assessment of climate variability in water resources systems**
- **Hydrologic predictions under different climatological conditions**

Challenges in Rainfall Simulation

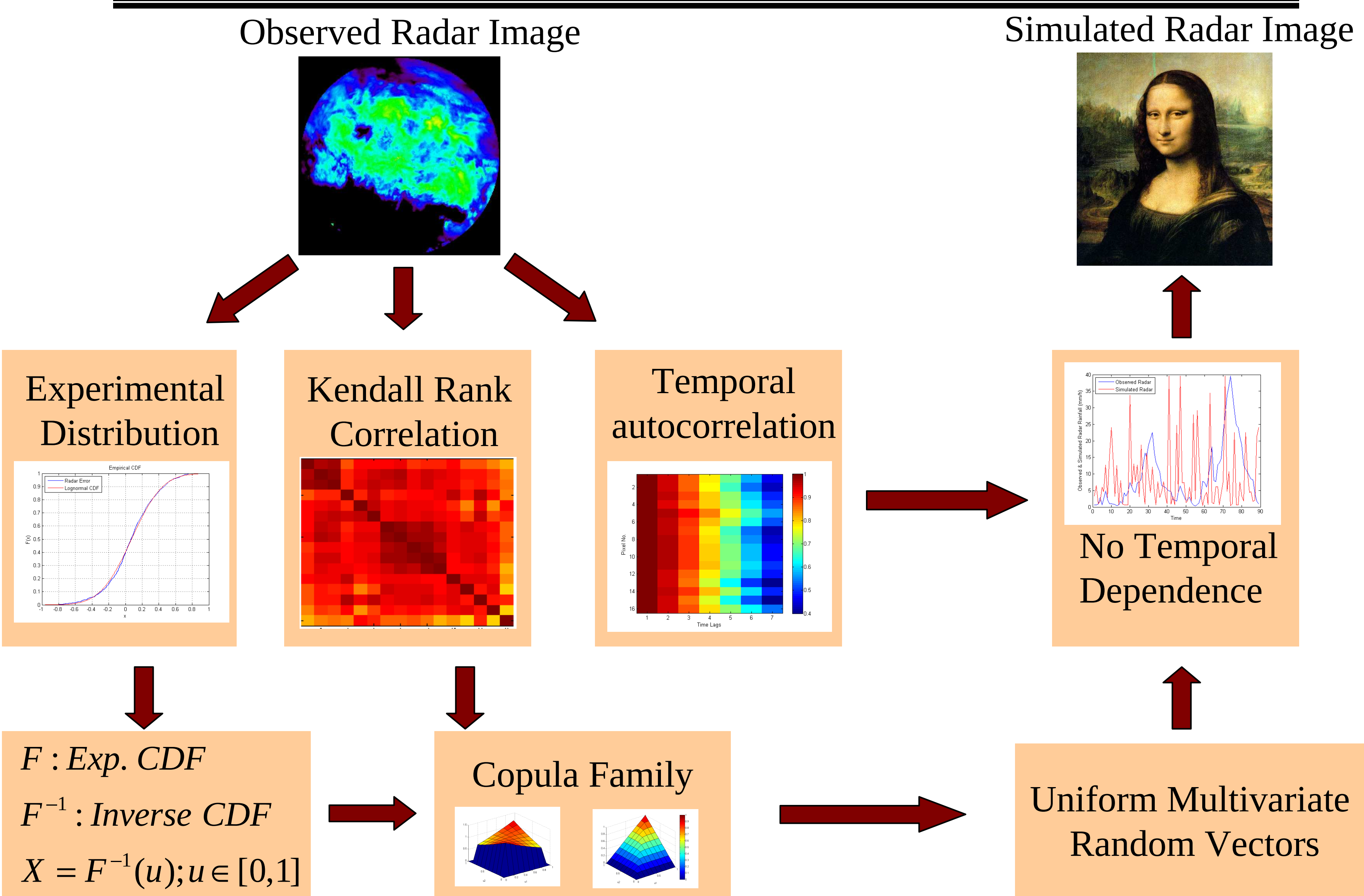
Most simulation technicqes are based on standard probability distributions. However, they may not be sufficiently flexible (Koutsoyiannis 2004, Moupfouma et al. 1982).

Rainfall distribution is strongly influenced by geographical, topographical and climatic changes (Suhaila 2007).

Two different rainfall types (e.g. convective, stratified) may belong to different distribution families.

Preserving temporal and spatial dependencies is not straightforward.

Methodology



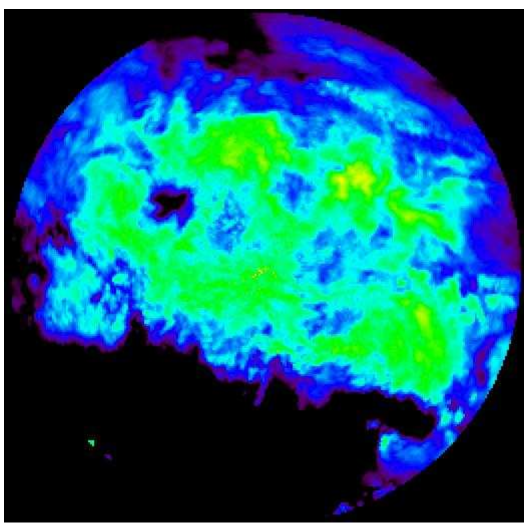
Amir Aghakouchak, András Bárdossy, Emad Habib

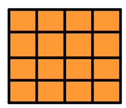
Study Area

State of baden wurttemberg
Germany

Temporal Resolution: 5 min
Spatial Resolution: 1 km^2

Simulation Area: 2500 km^2





Amir Aghakouchak, András Bárdossy, Emad Habib

Copulas

Multivariate distribution with uni-variate margins being u[0,1]

$$C(u_1,\ldots,u_n) = \Pr(U_1 \le u_1,\ldots,U_n \le u_n)$$

$$u_1,\ldots,u_n \in [0,1]$$

C : Joint cumulative distribution function of a random vector U whose marginals are $u[0,1]$

$$F(x_1,\ldots,x_n) = C(F_1(x_1),\ldots,F_n(x_n))$$

$$C(u_1,\ldots,u_n) = F(F_1^{-1}(u_1),\ldots,F_n^{-1}(u_n))$$

F : CDF with marginals $F1,\ldots Fn$ That could belong to even different distribution family

Applying a copula to univariate marginals results in a proper multivariate *PDF* that contains the information about the dependence structure of its components.

Gaussian Copula

Constructed from the bivariate normal distribution

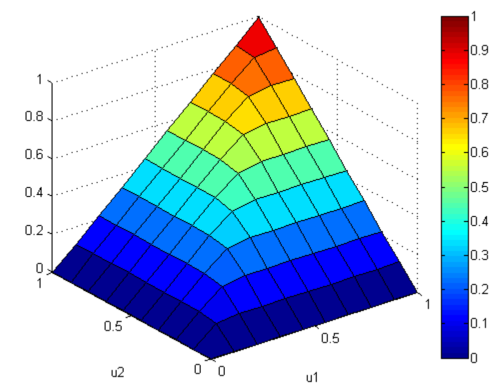
$$C_\rho(u_1,\ldots,u_n) = F_\rho^n(F^{-1}(u_1),\ldots,F^{-1}(u_n))$$

F: Gaussian CDF

F^n: Multivariate Gaussian CDF

ρ: Correlation matrix

Gaussian Cumulative Distribution Function

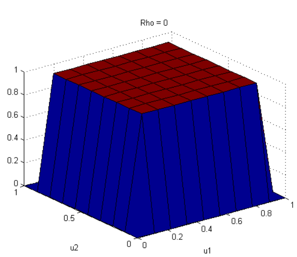


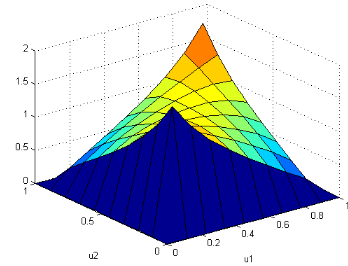
$\rho = 0$

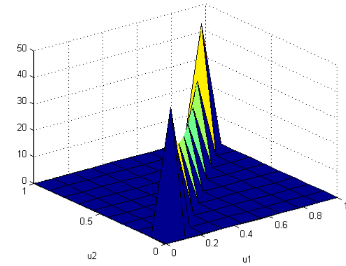
$\rho = 0.5$

$\rho = 0.9$

PDF







t-Copula

$$C_{\nu\rho}(u_1,\ldots,u_n) = t^n_{\nu\rho}(t_\nu^{-1}(u_1),\ldots,t_\nu^{-1}(u_n))$$

t: Student CDF

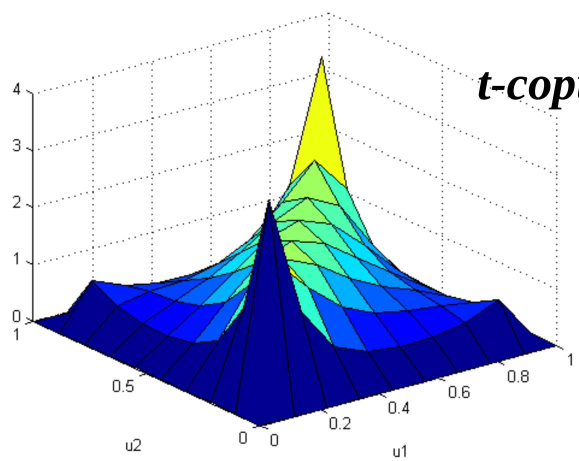
t^n : Multivariate Student CDF

ρ: Correlation matrix

ν: Degrees of freedom

t-copula shows a positive tail dependence that results in larger extreme events are generated simultaneously by t-copula than by Gaussian copula.

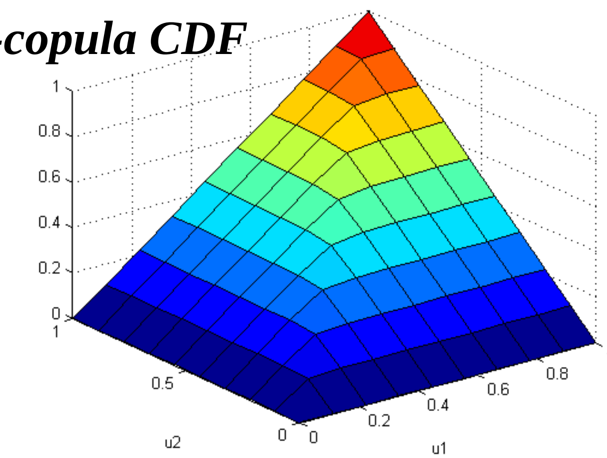
t-copula PDF



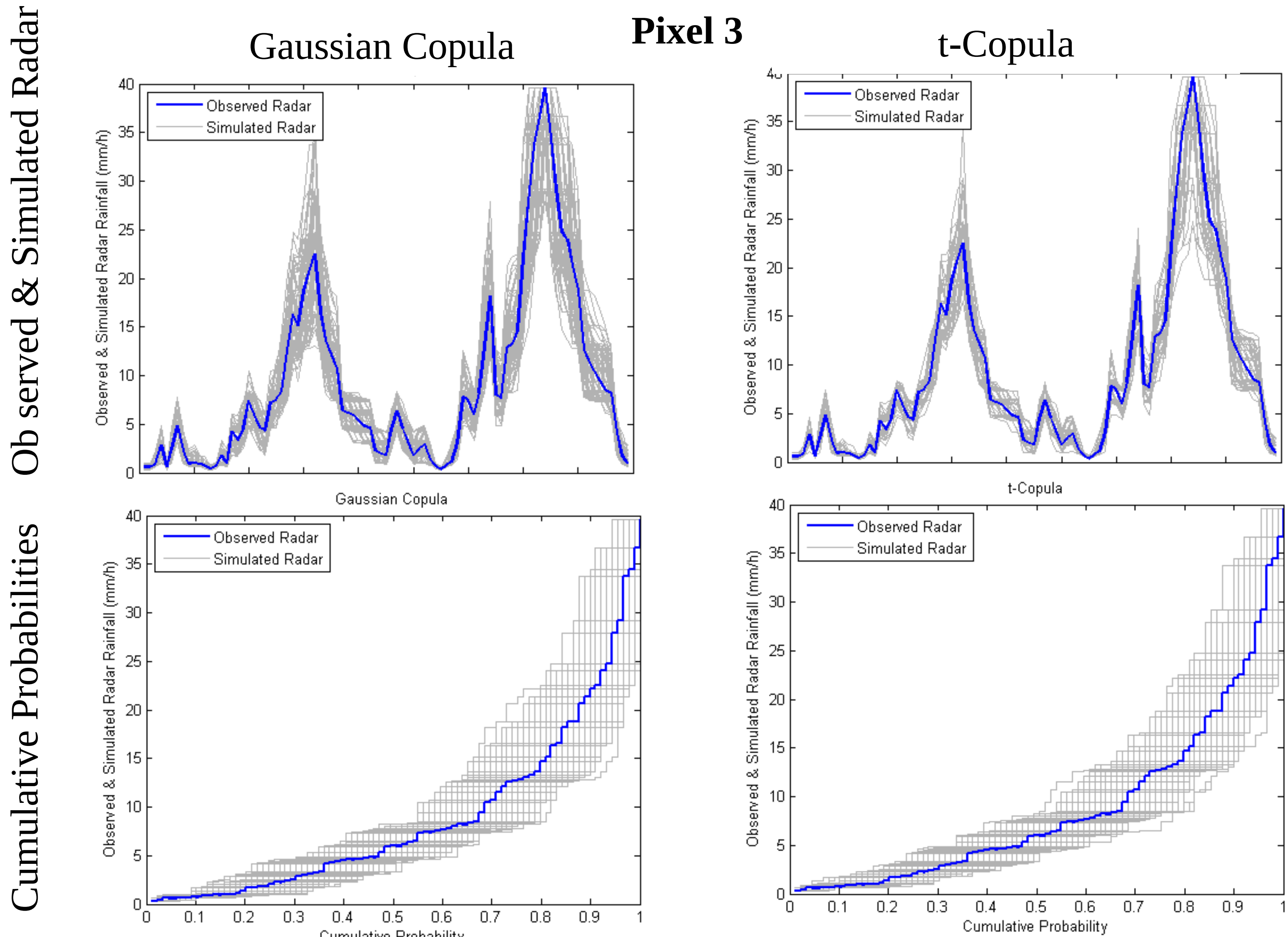
$\rho = 0.5$

$\nu = 1$

t-copula CDF



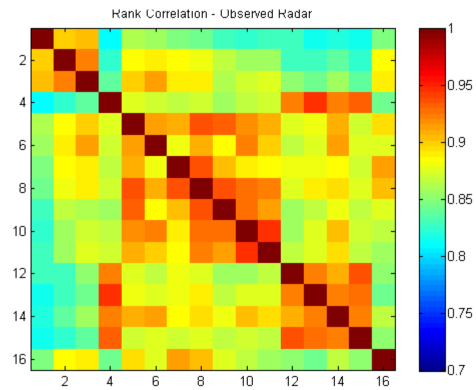
Simulation Using Gaussian t-Copula



Amir Aghakouchak, András Bárdossy, Emad Habib

Simulation Using Gaussian t-Copula

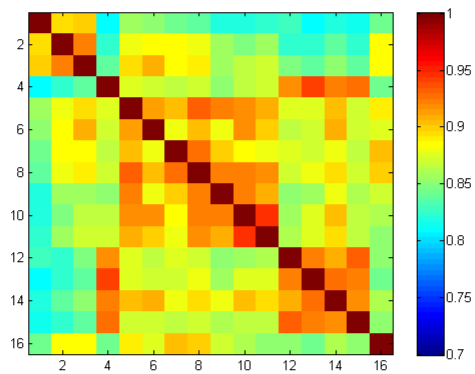
Spatial Corr. **Observed** Radar



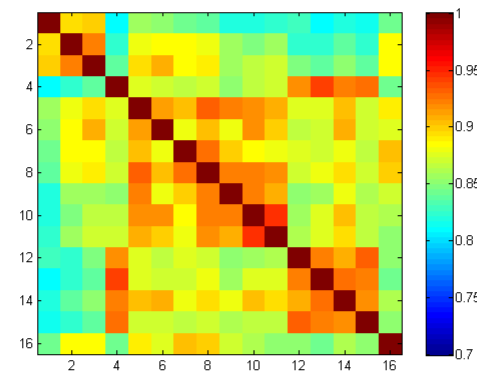
Kendall Correlation

$$\tau_{ij} = \binom{n}{2}^{-1} \sum \sum_{u<v} sign(x_u^i - x_v^j).sign(x_u^j - x_v^j)$$

Gaussian Copula

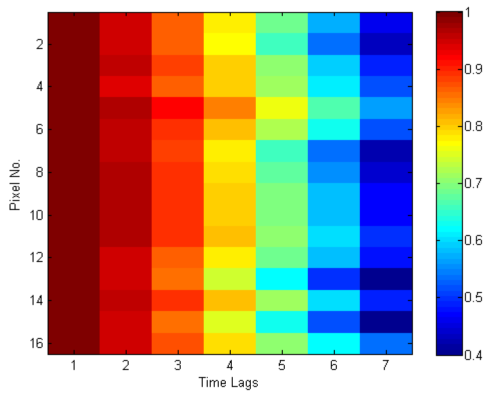


t-Copula

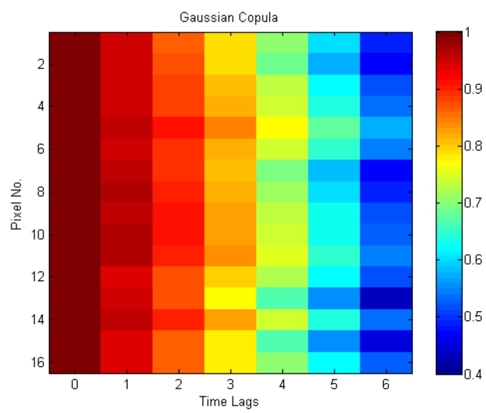


Simulation Using Gaussian t-Copula

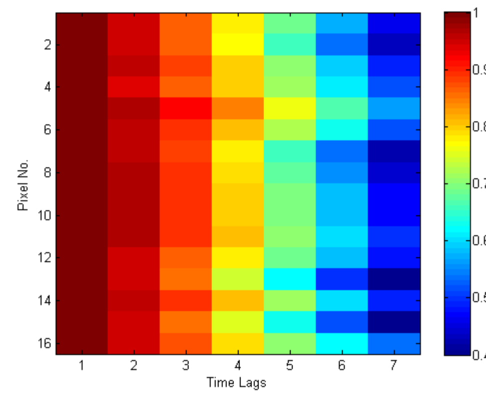
Temporal autocorr. **Observed** Radar



Gaussian Copula



t-Copula

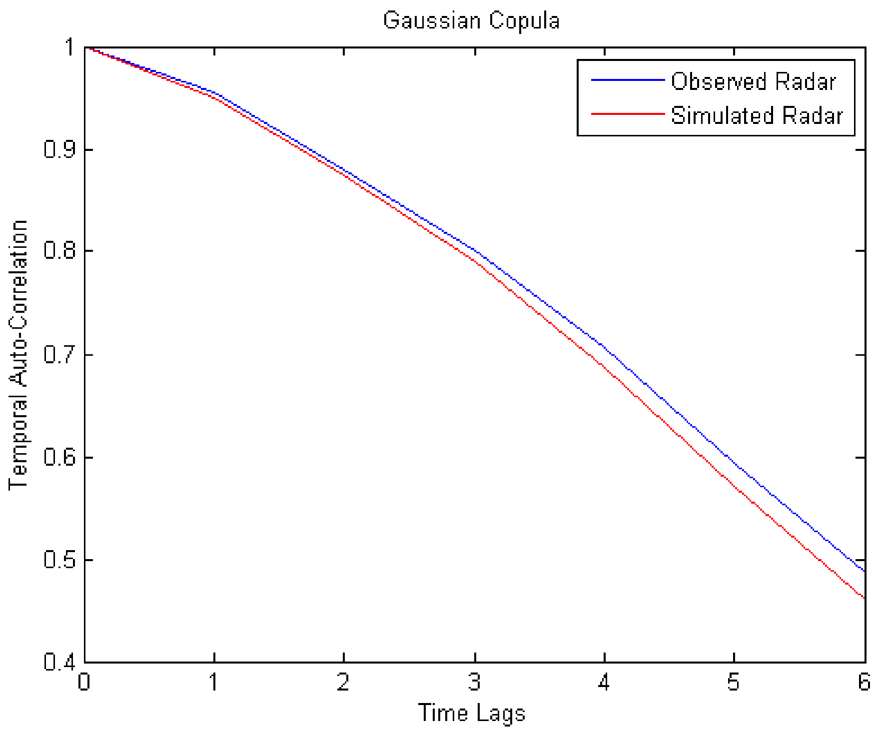


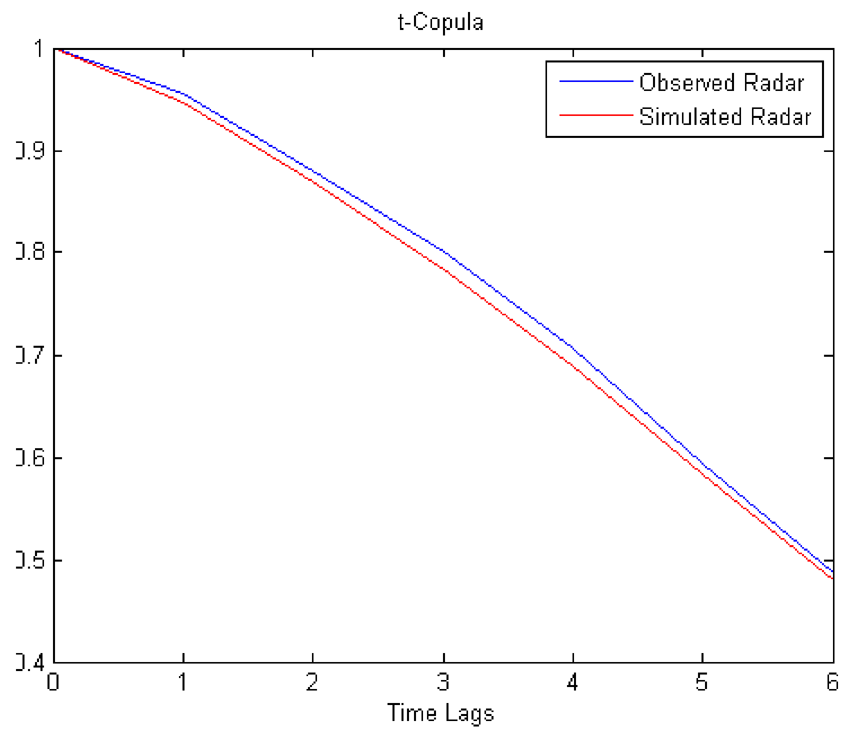
Simulation Using Gaussian t-Copula

Temporal Autocorrelation over Pixel 3

Gaussian Copula

t-Copula





Amir Aghakouchak, András Bárdossy, Emad Habib

Archimedean Copulas – Frank & Clayton

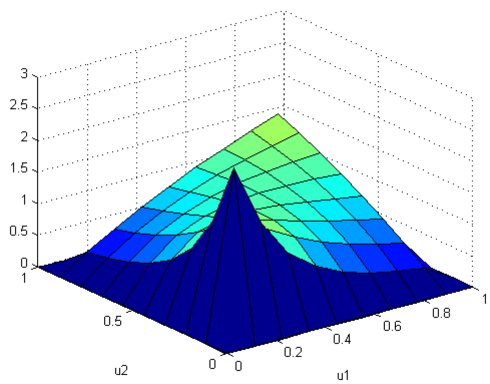
Archimedean Copulas:

- Clayton Copula
- Frank Copula
- Gumble Copula

$$C(u_1,\ldots,u_n) = \Psi^{-1}\left[\sum_{i=1}^{n}(F_i(x_i))\right]$$

Clayton Copula:

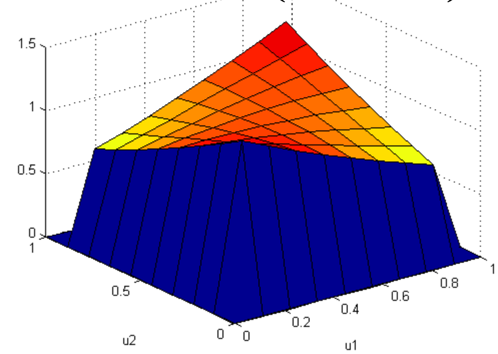
$$\Psi(x) = x^{\theta} - 1$$



Clayton PDF $\theta = 0.9$

Frank Copula:

$$\Psi(x) = \ln\left(\frac{e^{\theta x} - 1}{e^{\theta x}}\right)$$



Frank PDF $\theta = 0.9$

Archimedean Copulas – Gumbel

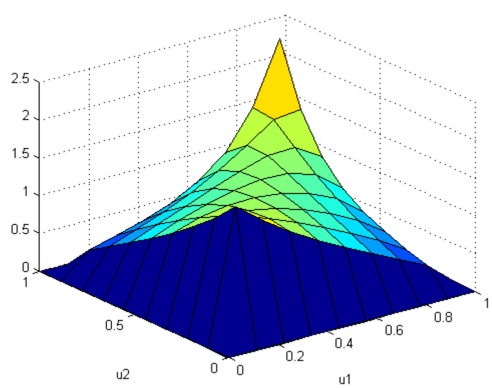
$$C(u_1,\ldots,u_n) = \Psi^{-1}\left[\sum_{i=1}^{n}(F_i(x_i))\right]$$

$$\Psi(x) = (-\ln(x))^{\theta}$$

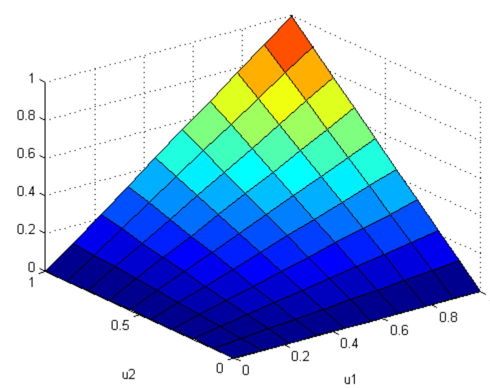
$$C(u_1,\ldots,u_n) = \exp\left[-\left[\sum_{i=1}^{n}(-\ln(u_i))^{\theta}\right]^{1/\theta}\right]$$

$$\theta > 1$$

Gumbel copula is very different with previously describes copulas as it can only model positive dependence structure.



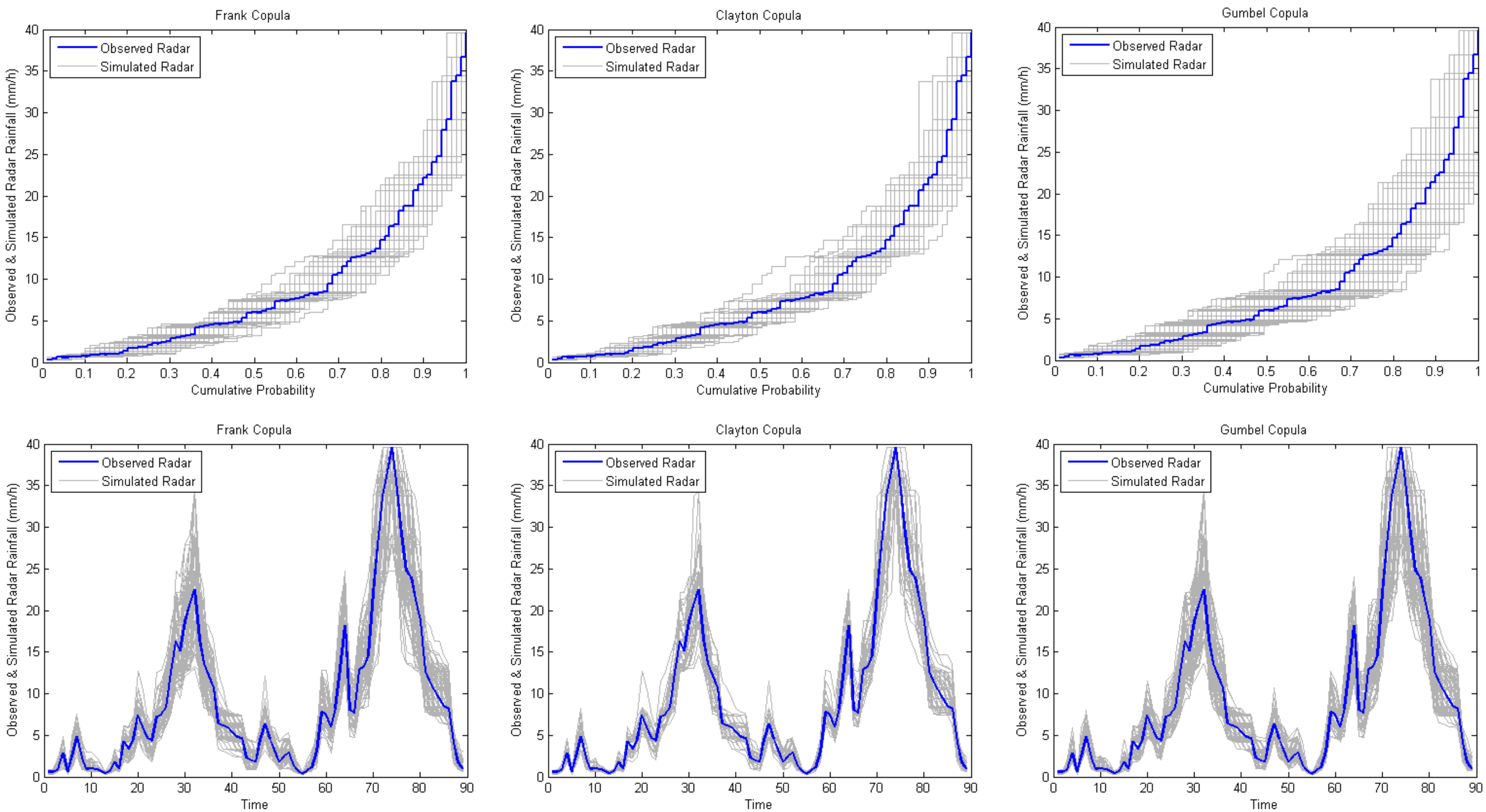
Gumbel PDF $\theta = 1.5$



Gumbel CDF

Frank, Clayton and Gumbel Copulas

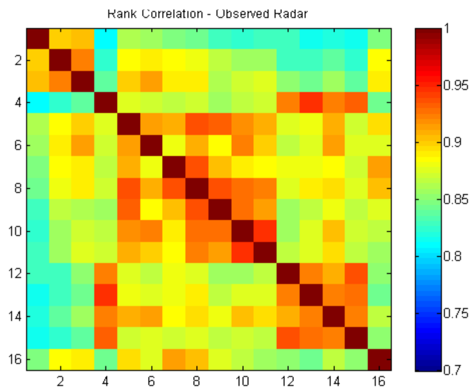
Frank Copula | Clayton Copula | Gumbel Copula



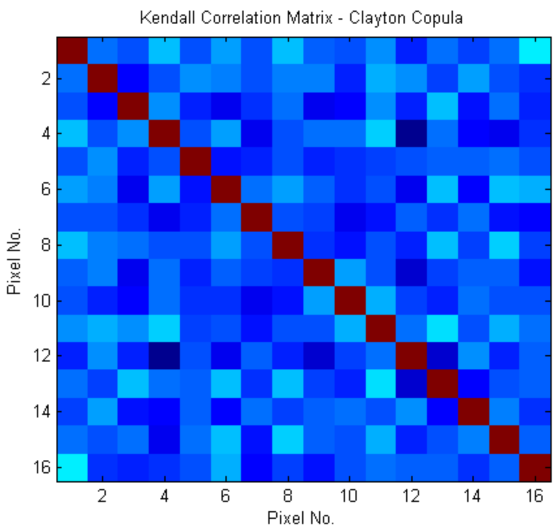
Amir Aghakouchak, András Bárdossy, Emad Habib

Simulation Using Gaussian t-Copula

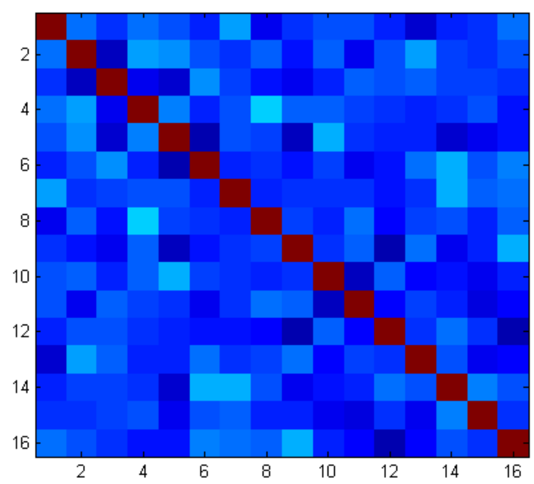
Spatial Corr. **Observed** Radar



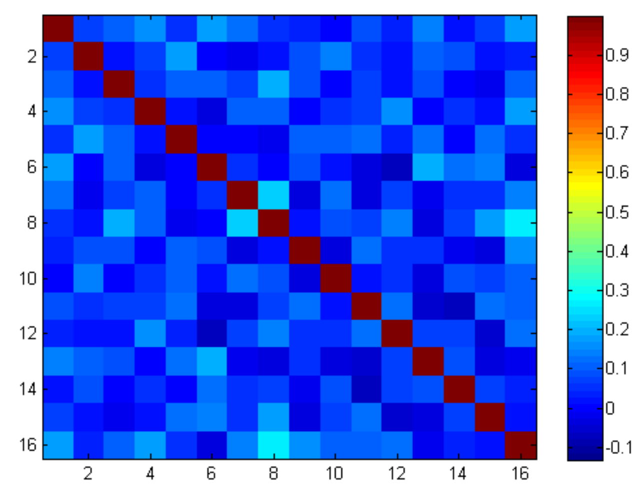
Clayton Copula



Frank Copula



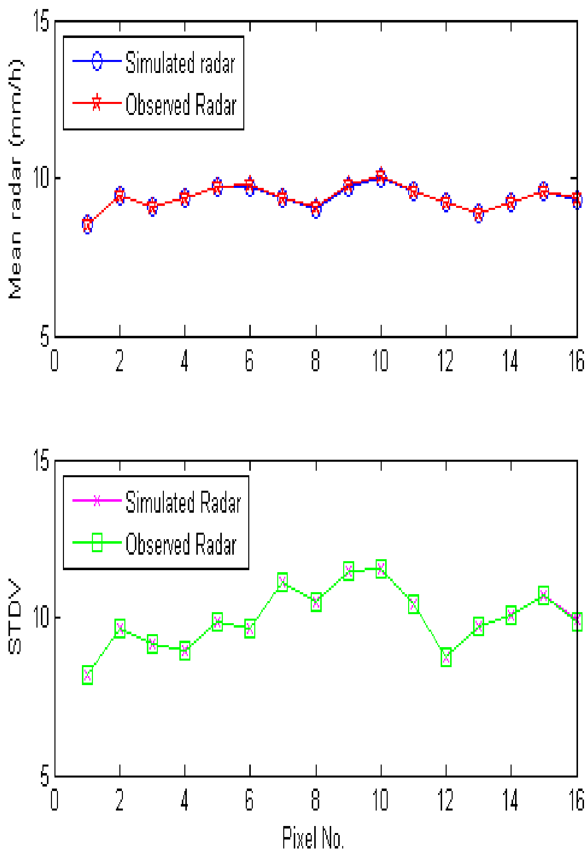
Gumbel Copula



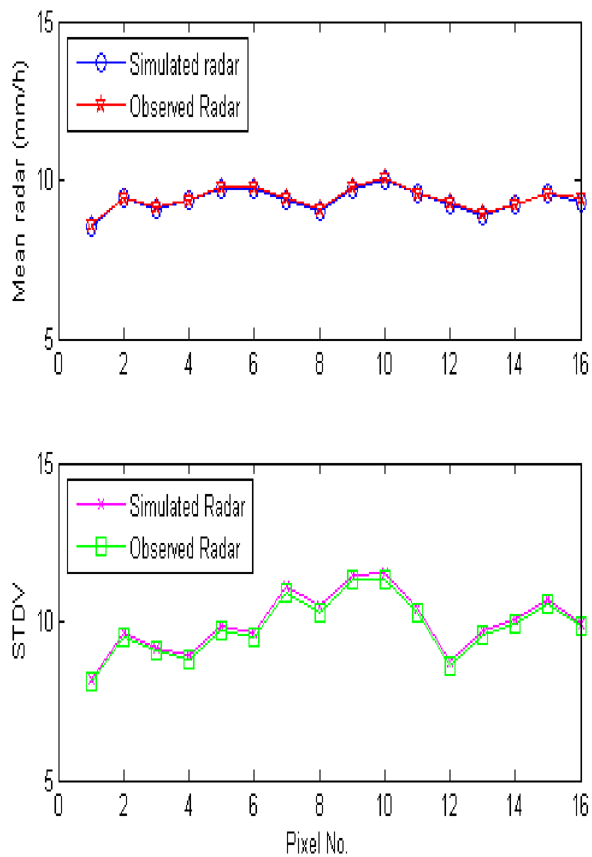
Amir Aghakouchak, András Bárdossy, Emad Habib

Frank, Clayton and Gumbel Copulas

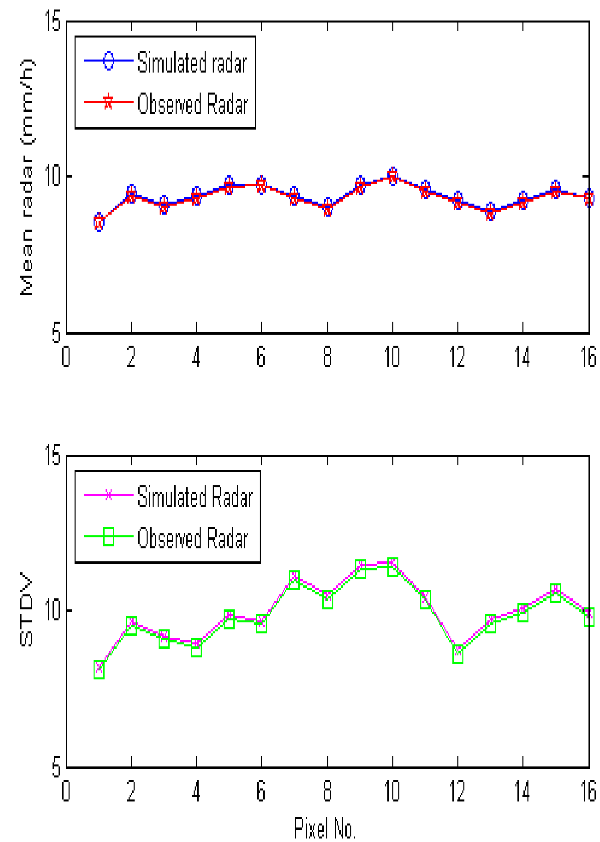
Frank Copula



Clayton Copula



Gumbel Copula



Amir Aghakouchak, András Bárdossy, Emad Habib

Comparison between Copula Families

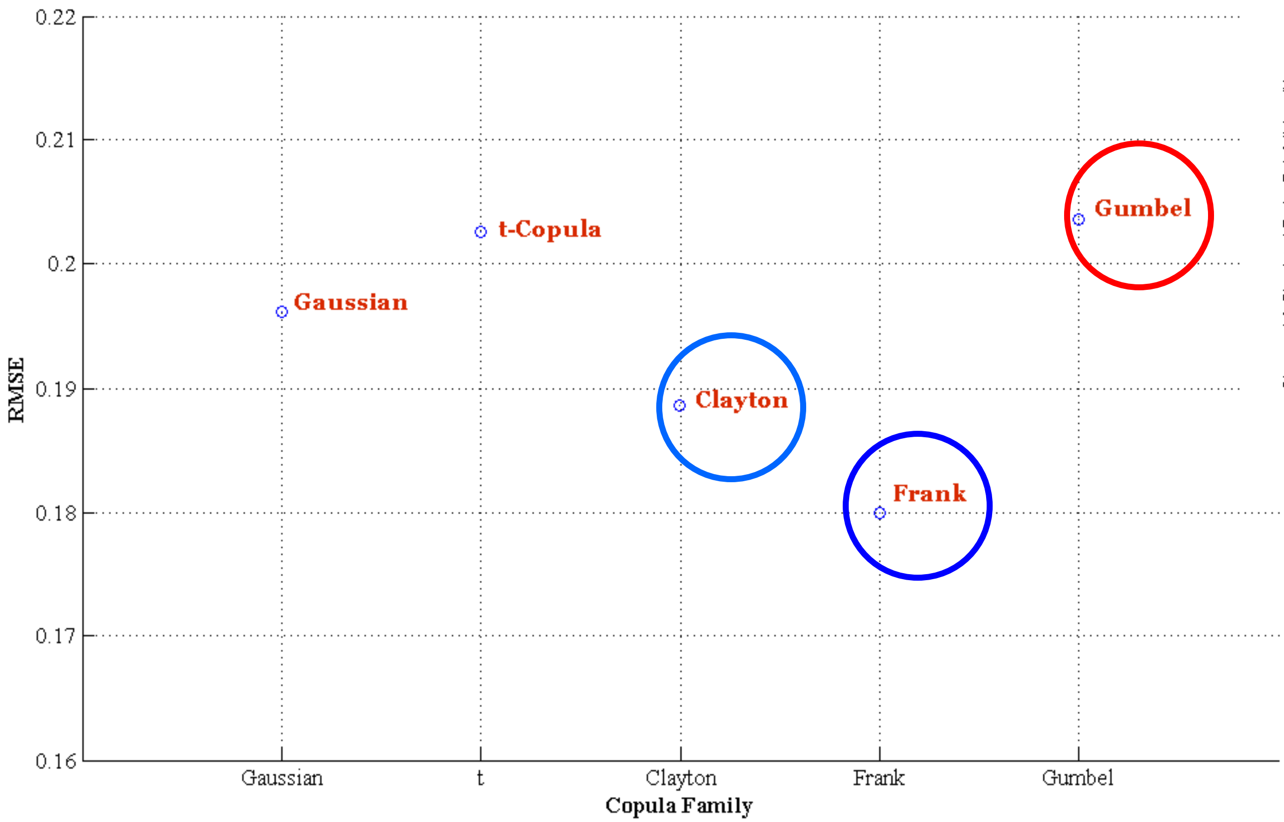
A comparison between spatial correlation of simulated and observed radar data in 500 simulations

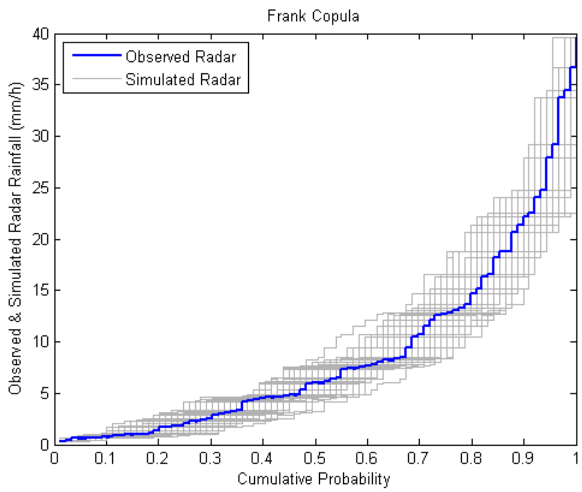
$$RMSE = \sqrt{\frac{1}{m-2}\sum_{j=1}^{m}\left[\frac{R_j - (Z_j / A)^{1/b}}{R_j}\right]^2}$$

Copula Family	RMSE
Gaussian	0.11
t-copula	0.05
Clayton copula	10. 11
Frank copula	10.97
Gumbel copula	9.21

Comparison between Copula Families

A comparison between cumulative probability functions of simulated radar data in 500 simulations





$$RMSE = \sqrt{\frac{1}{m-2}\sum_{j=1}^{m}\left[\frac{R_j - (Z_j / A)^{1/b}}{R_j}\right]^2}$$

Summary & Conclusions

Copula based simulation can over come typical challenges in multivariate simulations such as preserving spatial and temporal dependencies.

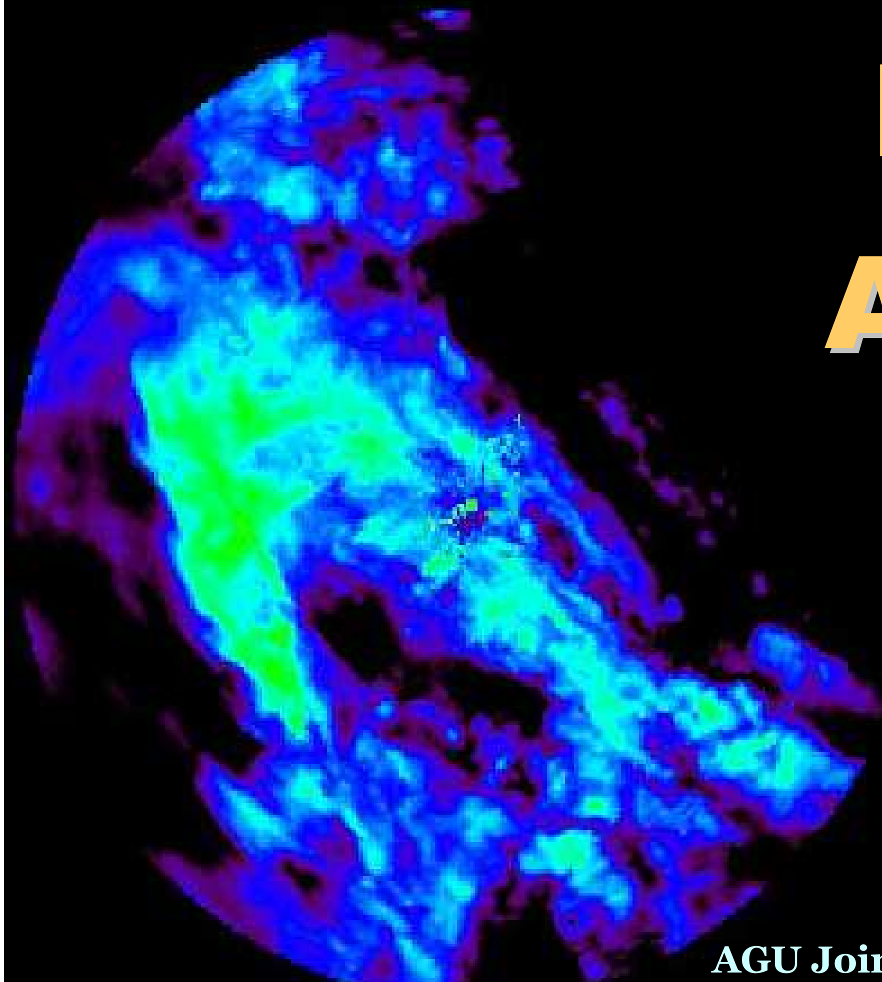
Simulation based on experimental distribution results in more reasonable simulated fields in terms of probability distribution.

t-copula seems to be the most appropriate copula family in terms of preserving spatial dependencies.

Frank, Clayton and Gumbel copulas fail to describe spatial dependencies in higher dimensions.

The most appropriate copula family with respect to extremes and their dependencies is still to be investigated.

THANK YOU FOR YOUR ATTENTION



Comparison between Copula Families

A comparison between temporal autocorrelation of simulated and observed radar data