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A method for filtering out raingauge representativeness errors from the verification distributions of radar and raingauge rainfall

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Abstract

The study presents a conditional distribution transformation (CDT) method for improving radar rainfall (RR) verifications that use sparse raingauge networks as the ground reference (GR). Large differences between the sampling areas of radar and raingauge measurements render direct comparisons problematic. The purpose of the CDT method is to filter out the raingauge representativeness errors from radar–raingauge verification samples. Our objective is to test the validity and evaluate the accuracy of this method. These analyses are based on two large data samples from high-density research networks covering the Goodwin Creek watershed in Mississippi and the Little Washita watershed in Oklahoma. An example implementation in a quasi operational situation is also presented, and sample size requirements are investigated using Monte Carlo simulations. Our tests indicate that the CDT method performs with satisfactory accuracy and can considerably improve on the currently applied RR verification practices.

Keywords: Radar rainfall; Raingauge; Area-point errors; Rainfall verification

1. Introduction

Despite the technological advances that have been made in the area of radar hydrology, quantitative characterization of the accuracy of radar-estimated rainfall amounts remains problematic. One of the difficult problems causing this situation is the inadequacy of the raingauge point measurements for reliable evaluation of the inherently area-averaged radar rainfall (RR) estimates. In this study, we discuss a new method to account for the raingauge representativeness errors in the process of comprehensive RR verification. We use the term ‘‘verification’’ to mean ‘‘evaluation,’’ or ‘‘assessment,’’ for consistency with the abundant literature on the verification of weather forecasts [19, for example].

The performance of all hydrological applications is highly dependent on the quality of rainfall estimates because rainfall is the main driving force behind the physical phenomena in hydrologic systems. Furthermore, modern prediction models often require explicit definition of the input errors. This is especially important for the ensemble hydrological forecasting [13]. Therefore, it is crucial that the RR products be equipped with comprehensive characteristics of the systematic and random discrepancies between RR estimates and the corresponding truth. Since radars provide estimates of surface rainfall averaged over certain spatial domains, the ground reference (GR) used to evaluate the estimates should match the corresponding true rainfall averaged over the same areas. With current technologies, accurate and systematic measurements of such ground truth are not available and the verification practices have to rely on comparing concurrent and collocated radar and raingauge rainfall accumulations.

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However, this has proven to be problematic due to the discrepancies between the spatial resolutions of the two sensors [2,5,7,12,15,21].

To account for the raingauge representativeness errors in the radar versus raingauge comparisons, Ciach and Krajewski [7] proposed an error variance separation (EVS) method whose purpose is to estimate the RR error variance devoid of the impact of the point-area differences caused by the spatial variability of rainfall. This approach was later used in several studies [1,6,15,32]. However, an empirical test by Ciach and Habib [9] revealed that the major assumption of the EVS method, i.e. negligible covariance between the radar and raingauge errors, is not fulfilled. Thus, the results obtained through using the EVS method can be inaccurate. In addition, the EVS is limited to one performance measure only, i.e. the error variance, which is inadequate to describe all the important aspects of RR uncertainties [8].

For complete statistical characterization of RR uncertainties at a given spatiotemporal scale and a given distance from the radar, it is sufficient to determine the bivariate distribution of RR estimates and the corresponding true values of rainfall averaged over the same spatiotemporal domain. This bivariate distribution, which we call the "true verification distribution" (TVD), is mathematically equivalent to a family of conditional distributions of the true areal rainfall conditioned on the values of the corresponding RR estimates. It enables estimating the functional–statistical structure of RR uncertainties [11], as well as a number of summary performance measures defined in the "distribution oriented" verification methodology [19]. Such distributions are usually not available because in most situations the only GR that can be used for verification of RR products comes from single-raingauge measurements. Large samples of concurrent and collocated radar and raingauge measurements allow the estimation of conditional raingauge rainfall distributions conditioned on the radar estimates. In the method presented here, we use such data, together with additional information about spatial variability of rainfall, to retrieve the conditional distributions of the corresponding true rainfall. These retrieved conditional distributions can then be applied to equip RR estimates with the probability distributions of their uncertainties. More generally, the method can be used to retrieve the TVDs from the broadly available radar–raingauge comparison data. In short, it can filter-out the raingauge representativeness errors from the typical RR verification samples.

In this study, we investigate a conditional distribution transformation (CDT) concept that is potentially an efficient way to achieve the aforementioned goal. The point-area transformation scheme used in the CDT method was first proposed by Journel and Huijbregts [18]. Later, it was expanded by Morrissey [25], and Morrissey and Greene [26], to the full conditional setup discussed in this study, and applied to the verification of satellite rainfall products. Based on a simulation experiment, they evaluated the performance of their procedure and reported satisfactory results. However, any conclusions based on simulations are strongly dependent on the underlying assumptions and, often crude, simplifications of the true processes. In this study, we present empirical tests of the CDT scheme that validate the proposed methodology.

2. Distribution-oriented verification of RR products

Concise and comprehensive assessment of RR estimates is not a trivial task. An efficient verification technique has to describe different quality aspects of the products in terms of a few key criteria. The distribution-oriented verification methodology [19] addresses this goal in a systematic manner. It derives all the product performance measures from the TVD of the corresponding RR and true rainfall values:

$$(R_a, R_r) \sim f(R_a, R_r), \tag{1}$$

where the symbol "$\sim$" denotes relation "distributed as", $f(\cdot,\cdot)$ is a bivariate probability density function, R_r and R_a are random variables representing the corresponding radar estimated and true rainfall values. The performance measures that are the most important for the verification of RR products are:

1. *Overall bias* that describes the systematic discrepancies between RR and the truth. It can be defined as the following multiplicative factor:

   $$B_0 = E\{R_a\}/E\{R_r\}, \tag{2}$$

   where $E\{\cdot\}$ is the operator of mathematical expectation. This statistic is often applied to as a multiplicative bias correction factor to remove the overall biases from the radar–raingauge samples [5,24,30].
2. *Mean square error* that describes the overall level of random differences:

   $$\mathrm{MSE} = E\{(R_r - R_a)^2\}, \tag{3}$$

   which has been the most common criterion used for the assessment and optimization of RR estimates [1,5–7,15,32, for example].
3. *Conditional biases* that describe the average departures of one variable from fixed values of the other variable. The *type 1* and *type 2* conditional biases can be defined as:

   $$\mathrm{CB}_1 = E_r\{(E_a\{R_a \mid R_r\} - R_r)^2\}, \tag{4a}$$

   $$\mathrm{CB}_2 = E_a\{(E_r\{R_r \mid R_a\} - R_a)^2\}, \tag{4b}$$

   where $E\{\cdot \mid \cdot\}$ is the operator of conditional expectation. These statistics are practically unknown in radar hydrology, although the presence of the conditional biases in RR products has been qualitatively described long ago [2]. However, their better understanding and systematic usage can substantially improve on the conclusive assessments of the products [8,11,19].

The above basic performance criteria, as well as several other possible measures, are rigorously defined in terms of the TVD defined by Eq. (1). However, the basic prerequisite to apply this framework is that, for a specified spatiotemporal scale, the TVD is either known with sufficient accuracy, or can be retrieved from the observational data. In this study, we test a method to retrieve the TVDs from the typical radar–raingauge comparison data.

3. Point-area distribution transformation method

Our implementation of the point-area transformation scheme follows in principle the methodology presented in Morrissey [25], and Morrissey and Greene [26]. Thus, only a brief description of the mathematical apparatus is presented here. Assume the second order stochastic homogeneity of rainfall within a spatial domain of area A. That means that the point rainfall mean, variance and spatial correlation function are the same within this spatial domain. Let R_p represent point rainfall with mean $E\{R_p\}$ and variance $\mathrm{Var}\{R_p\}$, and R_a represent the rainfall averaged over an area A with mean $E\{R_a\}$ and variance $\mathrm{Var}\{R_a\}$. Due to the homogeneity, the means of the two corresponding processes are equal, i.e. $E\{R_a\} = E\{R_p\}$. On the other hand, the variances can be related to each other based on the spatial correlation in the rainfall field in the following way [4]:

$$\mathrm{Var}\{R_a\} = \frac{\mathrm{Var}\{R_p\}}{A^2} \int_A \int_A \rho(x,y)\,\mathrm{d}x^2\mathrm{d}y^2. \tag{5}$$

Now, given the probability distribution of R_p, we want to estimate the distribution of R_a that has the same mean as R_p, but different and known variance. This problem, of course, cannot be solved without additional assumptions. One way to approach it can be based on the assumption that probability distributions of point and areal rainfall have the same shape in some predetermined sense. For example, several studies [20,29] have suggested that rainfall can be modeled using parametric distributions, such as lognormal or gamma, that are governed by two parameters only. If such distributional assumptions are valid, the transformation from point to areal rainfall distribution can be performed based on the relation between the first and second statistical moments described above. However, different parametric distributions can yield different results and, based on the literature, one cannot select the model that could apply to all spatiotemporal scales and precipitation systems. Also, practically no empirical data exist on the actual distributions of area-averaged rainfall in different situations. Thus, to avoid any specific parametric assumptions about the underlying distributions of both variables, we adopt a more general transformation method proposed by Journel and Huijbregts [18].

The probability distribution of raingauge measurements R_p can always be represented using a transformation that expresses R_p as a function of the standard normal random variable $R_p = \phi_{R_p}(u)$, where u is the standard Gaussian variable and the equality is in the sense of the same probability distributions. This function is approximated using a decomposition (expansion) based on Hermite polynomials:

$$\phi_{R_p}(u) \approx \sum_{i=0}^{n} \frac{\psi_i}{i!} H_i(u), \tag{6a}$$

$$H_0(u) = 1, \quad H_1(u) = -u, \quad H_2(u) = u^2 - 1,$$
$$H_3(u) = -u^3 + 3u, \tag{6b}$$

where $H_i(\cdot)$ are Hermite polynomials of the order i and ψ_i are their expansion coefficients, and the first four Hermite polynomials are shown as an example. The decomposition coefficients are fitted to the empirical frequency distribution of R_p using an iterative procedure described in Journel and Huijbregts [18]. The accuracy of the approximation depends on the number of terms, n, in the expansion (6) that can be selected to achieve satisfactory precision within reasonable computation time. The coefficients ψ_i are related to the mean and variance of the point rainfall as follows:

$$\psi_0 = E\{R_p\}, \tag{7a}$$

$$\sum_{i=1}^{n} \frac{\psi_i^2}{i!} = \mathrm{Var}\{R_p\}. \tag{7b}$$

The main assumption of the point-area transformation scheme proposed by Journel and Huijbregts [18] is that the function ϕ_{R_a}, expressing the areal rainfall as a function of the standard normal random variable (just as ϕ_{R_p} represents the point rainfall), has the same Hermite expansion as ϕ_{R_p}, but its decomposition coefficients are modified by a single scaling factor, a, in the following way:

$$\phi_{R_a}(u) \approx \sum_{i=0}^{n} \frac{\psi_i a^i}{i!} H_i(u), \tag{8}$$

where the coefficients ψ_i are the same as in (6). Note that the distribution transformation imposed by (8) preserves the distribution mean since $a^0 = 1$. On the other hand, the variance of the transformed distribution of the areal rainfall can now be expressed as:

$$\mathrm{Var}\{R_a\} = \sum_{i=1}^{n} \frac{\psi_i^2 a^{2i}}{i!}, \tag{9}$$

and thus, it depends on the known decomposition coefficients ψ_i of the raingauge rainfall and the scaling factor, a, only. Eq. (9) is a monotone function of a. Thus, if the variance of R_a is known, the scaling factor can be determined, using any iterative or graphical method, so that the equality (9) is fulfilled.

Given the estimates of the coefficients, ψ_i, and the scaling factor, a, the computer generated standard normal deviates can be substituted into (8) to simulate the distribution of the areal rainfall. The frequency distribution of the modeled values of R_a preserves the mean and variance of the true areal rainfall. It remains to be tested how other characteristics, like the distribution quantiles, are reproduces by this procedure.

The point-area transformation procedure that we outlined above is general. It can be applied to the whole data sample, as well as to its sub-samples selected (conditioned) in any specific way. Since our focus in this study is on verification of RR products, the distributions and their transformation have to be conditioned on the radar estimates, R_r.

The scheme of this conditional distribution transformation (CDT) can be summarized as follows. First, the raingauge rainfall values in the data sample are grouped into sub-samples that are conditioned on a number of ranges of the RR values, $(R_p|R_r = r)$, each range centered on a RR value, r. The number of the sub-samples and their sizes depend on the amount of available data. Then, the correlation function of the point rainfall conditioned on the radar estimate value, $(\rho|R_r = r)$, is estimated. This enables the estimation of the conditional variances of areal rainfall, $\mathrm{Var}\{R_a|R_r\} = r$. For each of the sub-samples $(R_p|R_r = r)$, the conditional coefficients, $(\psi_i|R_r = r)$, of the Hermite polynomial decomposition are estimated according to (6). Eq. (9) is then used to estimate the conditional scaling factors, $(a|R_r = r)$. Finally, the conditional distribution transformation functions $(\phi_{R_a}|R_r = r)$ are computed according to (8) and used to generate values that correspond to the areal rainfall $(R_a|R_r = r)$. These generated values can then be used to provide the desired estimates of the conditional distributions of the true area-averaged rainfall, $f(R_a|R_r)$, conditioned on RR. They can be used directly as statistical characteristics of the uncertainties in RR, or applied to reconstruct the TVD based on the following formula:

$$f(R_a, R_r) = f(R_a|R_r)f(R_r). \tag{10}$$

The TVD retrieved this way from a radar–gauge data sample can then be used in the distribution oriented verification procedures outlined in the previous section.

4. Data samples

The ideal data sample to empirically test the CDT method described above should include raingauge rainfall accumulations accompanied by the corresponding areal rainfall values averaged over the radar domains containing the raingauges. However, accurate area-averaged rainfall data are not easily available with the current rainfall measuring techniques. Fairly accurate approximations of the areal rainfall can be only obtained from super-dense networks of raingauges that cover uniformly a certain spatial domain [28]. Only two such data-sets were available to us for this study.

4.1. Goodwin Creek experimental network

The Goodwin Creek network covers a small experimental watershed located in northern Mississippi and has an area of about 21 km^2. Among various hydrometeorogical instruments, the watershed is densely covered with 34 raingauges as shown in Fig. 1. We used only 31 gauges due to data quality issues [30]. Most of the sites provide observation records that date back to 1981 and data from 20 warm seasons (April–October) were used in this analysis. In all the warm seasons, the monthly rainfall accumulations ranged from about 115–140 mm. Further details about the Goodwin Creek observations are available in Blackmar [3].

This dataset provides a unique opportunity to examine the proposed point-area transformation scheme over spatial and temporal scales that are close to the resolution of common radar products. We considered four time scales: 15 min, 1 h, 6 h and 24 h. Small time scales, such as 15 min and 1 h, are more relevant for RR studies, however, we also considered 6 and 24-h scales that are of interest for other hydrological applications. Due to the lack of matching radar data sample, we used the Goodwin Creek dataset to test the point-area distribution transformation scheme without conditioning on RR values.

4.2. Little Washita Micronet

This sample was created based on the same original data as those used in Ciach and Habib [9]. The Micronet

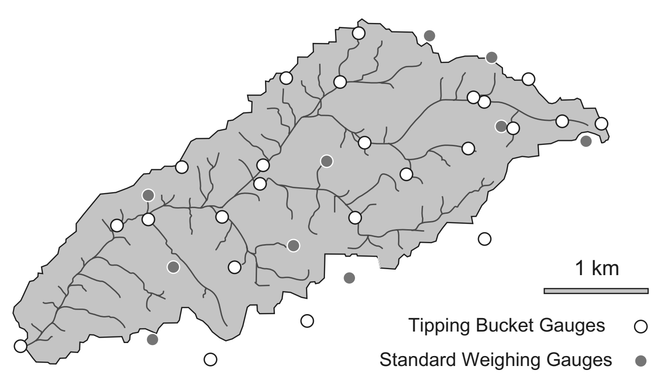

Fig. 1. A layout of the Goodwin Creek experimental watershed showing the locations of the 34 raingauges distributed within the testing area.

network is located in the Little Washita River watershed in Oklahoma and comprises a total of 42 stations over an area of about $1000\,\mathrm{km}^2$. However, to achieve uniform distribution of the raingauges in the analyzed areas, and to increase the data sample size, we selected three rectangular sub-domains of about 19×18-km^2 each within the watershed. A schematic layout of the areas and the gauge coverages are shown in Fig. 2. Each sub-domain is covered with 15–16 stations that provide satisfactory approximations of the area-averaged rainfall [9]. Obtaining the largest possible sample was the main concern of this analysis and therefore we allowed partial overlapping between the three areas.

Data of two warm seasons (April–September) of 1998 and 1999 were available for this study. A total rainfall accumulation of about 700 mm was collected over the watershed during this period. The raingauge data were supplemented with RR estimates averaged over the three sub-domains to test the point-area transformation method in a conditional setup (the full CDT scheme). The radar data are based on composite radar reflectivity maps produced by the NASAs Global Hydrology and Climate Center (GHRC). The maps are created from the "NOWrad©WSIs mosaic radar" products that are combined from individual radars of the NEXRAD operational network. They are provided in 16 levels of reflectivity and are available every 15 min with spatial resolution of about $2 \times 2\,\mathrm{km}^2$. The reflectivity-rainrate conversion was performed using a typical power-law Z–R relationship with the exponent set to 1.4 (standard NEXRAD). The Z–R multiplier was adjusted so that the sample averages of the areal rainfall estimates based on the radar and raingauge data are equal.

With only 12 months of observations, the sample size is relatively small. Therefore, in this study, we considered only one time scale of 15 min. Note also that the selected rectangular sub-domains are considerably larger than typical radar resolutions and that the single-raingauge representativeness errors are large in such a case. It makes the test of the raingauge-error filtering method presented here especially demanding.

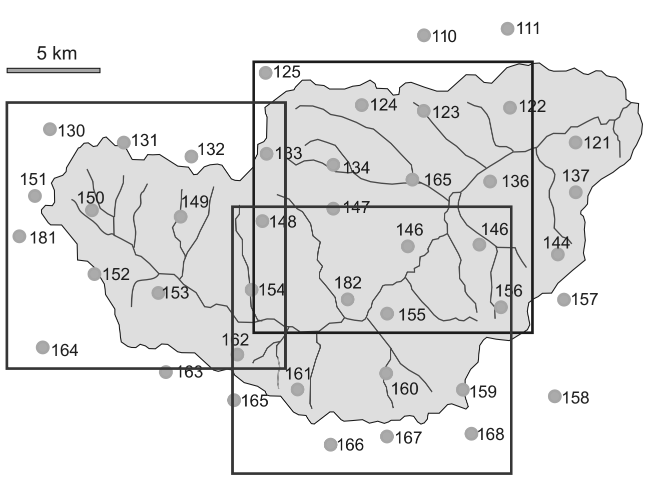

Fig. 2. A layout of the Little Washita Micronet with the three rectangular areas of about $19 \times 18\,\mathrm{km}^2$ that are used for testing the CDT method. Raingauges within each area provide approximations of the true areal rainfall.

5. Tests of the transformation scheme

The goal of the point-area transformation scheme is to obtain the probability distribution of areal rainfall based on the distribution of point rainfall and information on the spatial correlation in the rain-field. We begin with testing the proposed method in the unconditional setup using the large sample of raingauge data from the Goodwin Creek watershed. Then, we proceed with testing the full CDT method using the radar–raingauge sample from the Little Washita watershed. Below, we describe both tests and their results.

5.1. Unconditional transformation

To test the transformation scheme, we need first to consider the issue of the rainfall zero-intermittence. The probabilities of zero value for the point and areal rainfall are not equal, and zero areal rainfall implies no rainfall at each point of the area. Since eventually the distribution transformation scheme will be conditioned on positive values of RR estimates, the cases of zero areal rainfall will be of no practical interest. Additionally, we found out that the Hermite expansion (6) has difficulty dealing with them. Therefore, in constructing the bivariate point-area sample from the Goodwin Creek data, we limited it to the situations with the area-averaged rainfall, R_a, greater than zero.

Another point concerns the estimation of areal rainfall variance, $\mathrm{Var}\{R_a\}$. In actual applications of the transformation scheme, $\mathrm{Var}\{R_a\}$ has to be determined from the spatial correlation function according to (5). However, in testing the point-area transformation method, we estimated the variance directly from the actual areal rainfall values. By doing so, we avoid the effect of the uncertainties in estimation of the correlation function and focus our test on the validity of the point-area distribution transformation scheme itself. The procedure of evaluating the method's ability to reproduce areal rainfall distribution can be summarized as follows:

1. Construct the sample of concurrent point and areal rainfall for a specified spatiotemporal scale. The R_a values are approximated by averaging the raingauge observations within the area of interest, whereas the R_p values come from all the individual gauges.

2. Estimate the sample variances of the point and areal rainfall values. Estimate the Hermite expansion coefficients for the point rainfall and the value of the scaling coefficient.
3. Perform the distribution transformation procedure described in Section 2 to retrieve the areal rainfall distribution.
4. Compare the retrieved areal rainfall distribution against the observed one.

The scaling coefficient, a, estimated in the step 2 of the above procedure assumed values from 0.83 to 0.98, depending on the time-scale. Its value for the 24-h accumulation scale is close to one because the difference between point and areal rainfall in this case is small. To apply the point-area transformation procedure, one needs to make a choice about the number of terms in the Hermite expansions (6) and (8). In this study, $n = 40$ was used. According to our experience, the only trade-off for this choice is the computation time required to fit the expansion coefficients. Of course, larger n provide more accurate approximations, however, the improvement for $n > 40$ was insignificant.

The results of the test based on the Goodwin Creek data are shown in Fig. 3 for four time scales. In this figure, we compare the quantiles of the retrieved areal rainfall values, obtained using the transformation scheme, versus the quantiles of the observed values of R_a that correspond to the same percentiles (standard quantile–quantile plots). The comparisons show that the distributions agree quite well. Note that the last point in each plot corresponds to the 100% percentile, that is to the maximum areal rainfall in the sample for a given time scale. It is remarkable that the transformation scheme is able to reproduce the distributions of the extreme rainfall values with such a high degree of accuracy. In the same plots, we also show the corresponding quantiles of the observed point rainfall to demonstrate the differences between point and areal rainfall distributions. As expected, the differences are larger for higher values of rainfall and shorter time scales.

5.2. Conditional transformation

To evaluate the performance of the full CDT method, that is the point-area transformation scheme condi-

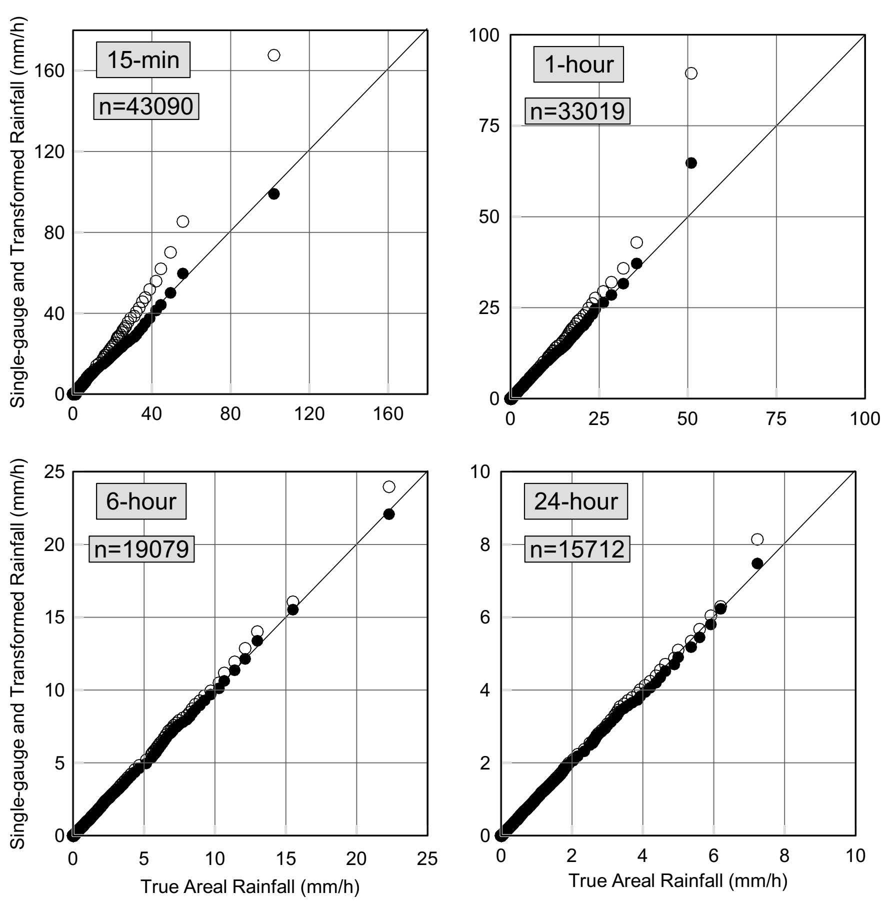

Fig. 3. A quantile–quantile plot of the cumulative rainfall distributions for the Goodwin Creek data. Filled circles correspond to the transformation-based versus the true areal rainfall distributions. Open circles correspond to the point (single-gauge) versus the true areal rainfall distributions. In the figure, n refers to the size of each sample.

tioned on radar estimates, we used the data sample of point rainfall, areal rainfall and the corresponding RR described in Section 3. Only one time scale of 15 min is considered here due to the limited size of this sample. First, we evaluated the distribution transformation method with a general condition of positive radar estimate, $R_r > 0$, in analogy to the unconditional test described in the previous section. The results of this analysis are presented in Fig. 4 in the same form of a quantile–quantile plot as in Fig. 3. Despite much smaller data sample, the agreement between the measured and CDT-retrieved areal rainfall distributions for the 15-min time scale is only slightly worse than in Fig. 3. The maximum rainfall values are smaller in this case because of the considerably larger averaging area.

Next, we proceeded with testing the full CDT scheme. We stratified the sample into sub-samples of four intervals of the 15-min RR values, R_r. Due to the sample size limitation we could consider only such a small number of the RR intervals. For each sub-sample separately, we carried out the same point-area transformation procedure as in the unconditional test described above. As before, we estimated the variances of the areal rainfall based on raingauge data so that the test is focused on the point-area transformation scheme only. The estimates of the scaling coefficient, a, assumed values of about 0.6, for each of the sub-samples (as well as for the whole sample conditioned only on $R_r > 0$). This value is significantly smaller than the value obtained for the Goodwin Creek sample because the averaging area and, consequently, the difference between the point and areal rainfall distributions is much larger in this case. The conditional quantile–quantile plots resulting from these tests are shown in Fig. 5. The plots indicate

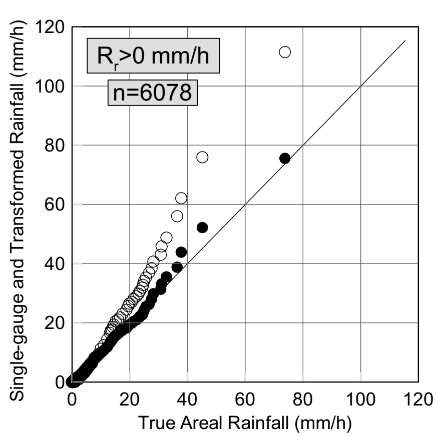

Fig. 4. A quantile–quantile plot of the cumulative rainfall distributions for the Little Washita data. Filled circles correspond to the transformation-based versus the true areal rainfall distributions. Open circles correspond to the point (single-gauge) versus the true areal rainfall distributions. The distributions are conditioned only on the RR values greater than zero. In the figure, n refers to the sample size.

that, overall, the transformation-based distributions are in a good agreement with the observed conditional distributions of areal rainfall. However, the degree of the agreement is not as high as in the case of the Goodwin Creek data, especially in the region of larger rainfall values. This is most likely due to the sampling errors caused by small sizes of the sub-samples. Nevertheless, the tests confirm that the CDT method is able to retrieve the conditional distributions of the areal rainfall with quite good degree of accuracy.

6. Implementation case study

We now apply the transformation-based approach to another data sample collected in a quasi-operational situation. This application demonstrates practical implementation problems and illustrates the capability of the CDT method to quantify the uncertainties of operational RR products based on data samples that are considerably smaller than those that we used in testing of the method. In this analysis we used the radar estimates that are the standard 2A-53 products available from the Ground Validation Program (GVP) of the Tropical Rainfall Measuring Mission (TRMM). These products represent "instantaneous" rain rate maps constructed from radar reflectivity volume scans recorded by the WSR-88D radar at Melbourne, Florida. These maps have the resolution of 2 km by 2 km and were constructed every 5–6 min. The GVP used a power-law Z–R relationship empirically fitted based on raingauge observations from several operational networks within the radar umbrella. A value of 1.4 was always used for the Z–R exponent, while the multiplier was selected to adjust the monthly accumulations of the radar pixels to corresponding raingauge accumulations averaged over all the available operational raingauges. Further details about the development of the 2A-53 products can be found in Marks et al. [24]. For this implementation study, we used raingauge data from the experimental Dense Rain Gauge Network (DRGN) that was operated during August and September of 1998 as part of the TExas FLorida UNderflights Experiment (TEFLUN-B). Fig. 6 shows the layout of this cluster. The DRGN network was not used by the GVP in developing the 2A-53 RR products and can be applied as an independent validation dataset. Further details about the characteristics of this radar–raingauge data sample can be found in Habib and Krajewski [15] and Datta et al. [12]. We point out that evaluating the accuracy of the 2A-53 radar products is not an objective of this analysis. Rather, these products are used to illustrate the application of the CDT method presented here.

For this implementation case we constructed a bivariate sample of radar and raingauge data by averaging the raingauge measurements over 5-min intervals

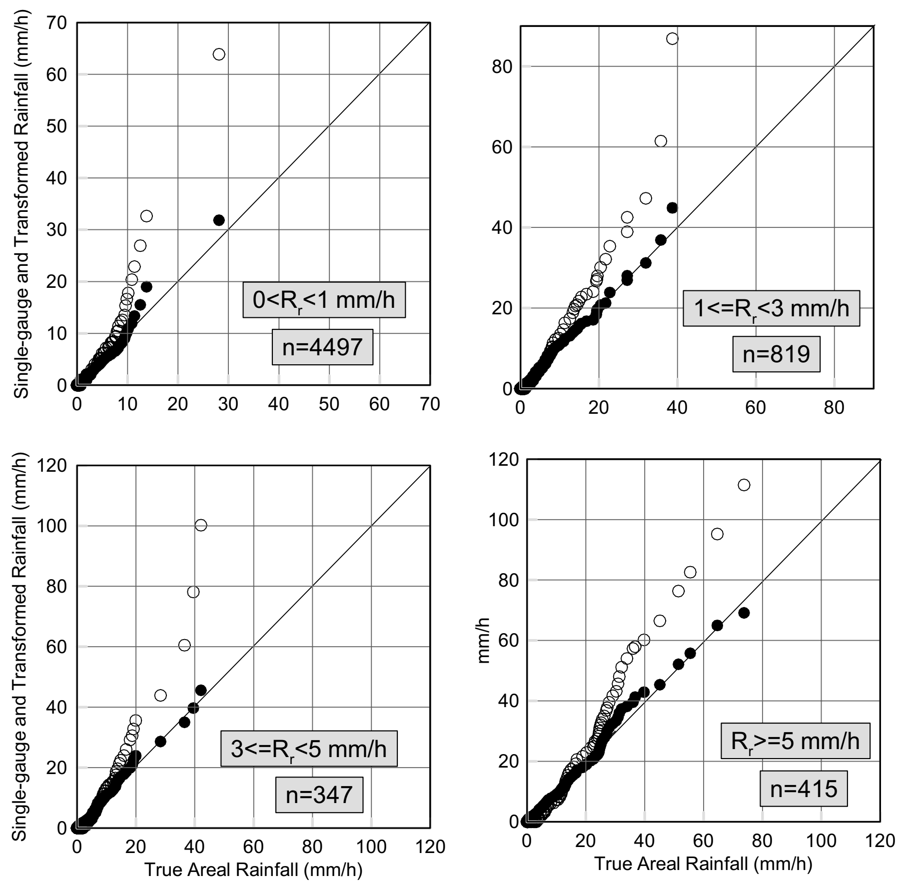

Fig. 5. A quantile–quantile plot of the cumulative rainfall distributions based on the same data as in Fig. 4, but conditioned on the radar rainfall. Filled circles correspond to the transformation-based versus the true areal rainfall distributions. Open circles correspond to the point (single-gauge) versus the true areal rainfall distributions. For this conditional analysis, the sample was stratified into four ranges of RR estimates. In the figure, n refers to the sample size in each range.

corresponding to the radar observation times. For such a short time scale, the rainfield decorrelation is substantial even at the distances of up to 2 km considered here [15]. We divided the sample into several sub-samples conditioned on the RR values as shown in Fig. 7. As indicated in this figure, we selected wider bins for higher radar estimates to maintain reasonable sample sizes. Due to the limited number of high rainfall values, we had to limit the analysis to $R_r \leqslant 50$ mm/h.

Next, we need information about the rainfield conditional correlation functions to estimate the variances of true areal rainfall in the sub-samples according to (5). We estimated the correlation functions from the inter-gauge correlation coefficients over a spatial scale of up to 2 km that corresponds to the resolution of radar maps under consideration. Due to the limited size of the TEFLUN-B sample, we estimated the correlation functions only in three ranges of the RR intensities: 0–2, 2–10 and above 10 mm/h. We estimated the inter-gauge correlation coefficients based on the lognormal transformation method proposed by Habib et al. [16] to reduce the biases and sampling errors caused by the inherent skewness of the rainfall distribution. We examined the lognormality assumption of this method by applying the normal plots and the Kolmogorov–Smirnov goodness of fit tests to the transformed raingauge data in the three radar conditioned sub-samples. The tests showed that the procedure can be used with these data. The degree of improvement was minor for the first two RR ranges. However, for the sub-sample with RR above 10 mm/h, the corrected correlation coefficients were smaller by up to 0.3 than the uncorrected estimates.

For each of the three ranges of RR intensities, we fitted an exponential function to the correlation coefficients:

$$\rho(d) = \rho_0 \exp\left(-\frac{d}{d_0}\right), \tag{11}$$

where d is the distance between any two points, $(\rho_0 - 1)$ is the instant decorrelation that can be caused by the random instrumental errors and d_0 is the correlation distance. The two correlation parameters, ρ_0 and d_0, were estimated for each of the three ranges of RR intensities. The plots of the correlation coefficients and the fitted

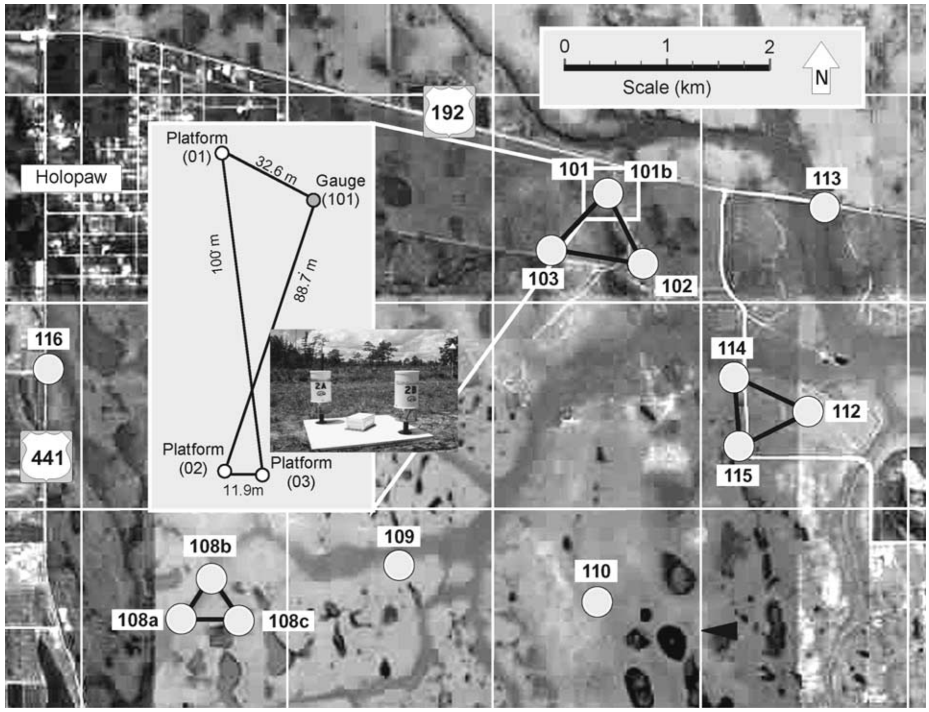

Fig. 6. An aerial photo of the dense raingauge network (DRGN) deployed during the TEFLUN-B field campaign in Florida in August–September of 1998. The $2 \times 2\,\mathrm{km}^2$ grid shows the pixels of the 2A-53 RR products. The circles show gauge locations with the numbering system that corresponds to the TRMM database. The insert shows one of the double-gauge platforms used in this experiment.

models with their parameters are presented in Fig. 8. It shows distinct differences in the correlation features between the three ranges of RR intensities. The conditional instant decorrelation, $(\rho_0 - 1)$, is larger for lower rain-rates, which is in agreement with the studies on the local uncertainties in tipping-bucket raingauges [10,17]. On the other hand, the conditional correlation distance, d_0, drops rapidly with increasing rainfall intensities and indicates high levels of spatial variability in the areas closer to the centers of the convective cells in Florida.

Note, that the selection of a specific model for the conditional correlation function is an implementation task and has to be driven by the actual behavior of the inter-gauge correlation coefficients. Eq. (6) and the CDT method as described in Section 3 are general and can use any suitable parametric, or nonparametric, correlation function. Also, one must be aware that any conditioning can affect the dependence structure of random variables. As a consequence, the conditional correlation functions might be quite different from the correlation between the unconditioned variables.

To proceed with the implementation of the CDT method, we carried out the following steps for each of the radar-conditioned sub-samples shown in Fig. 7. First, we computed the means and variances of the 5-min point rain-rates for each sub-sample. Next, based on the average radar rain-rate in each sub-sample, we selected the two parameters of the correlation function from the results presented in Fig. 8. The radar-conditioned variances of the areal rain-rates could then be evaluated using (5). Finally, we applied the point-area transformation scheme to generate the full distribution of the areal rain-rates for each of the sub-samples. We present the results of this procedure graphically in Fig. 9. The figure shows a family of conditional distributions of the 5-min areal rain-rates that was generated using the CDT procedure described above. These conditional distributions are functions of the RR intensities. We describe the distributions using the following percentiles of the radar-conditioned true areal rain-rates: 2.5%, 10%, 25%, 50%, 75%, 90%, and 97.5%. It is clear that the radar estimates are characterized by quite large uncertainty bounds. The range of these bounds increases with growing RR rates. For example, for the radar estimated value of about 10 mm/h, the 25–75% percentiles are from about 5–25 mm/h, whereas the 2.5–97.5% percentiles reach a broad range of about 0.4–29 mm/h. The uncertainty bounds become even larger for higher radar rain-rates. Note that the estimation of the uncertainty bounds for higher RR values can be subject to significant sampling errors due to small sample sizes.

7. Sample requirements

The RR uncertainty bounds estimated using the CDT method can be subject to significant sampling errors. To

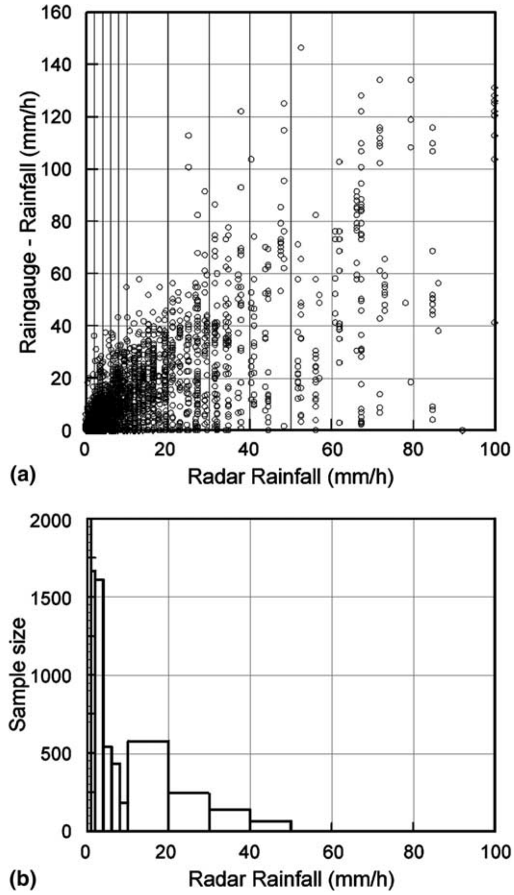

Fig. 7. Stratification of the Florida radar–raingauge data into several ranges of RR values used in the implementation case study of the CDT method. (a) A scatter plot of instantaneous $2 \times 2\,\text{km}^2$ RR estimates and the corresponding 5-min raingauge rain-rates with vertical lines indicating the ranges of RR. (b) The sub-sample sizes in each of the RR ranges.

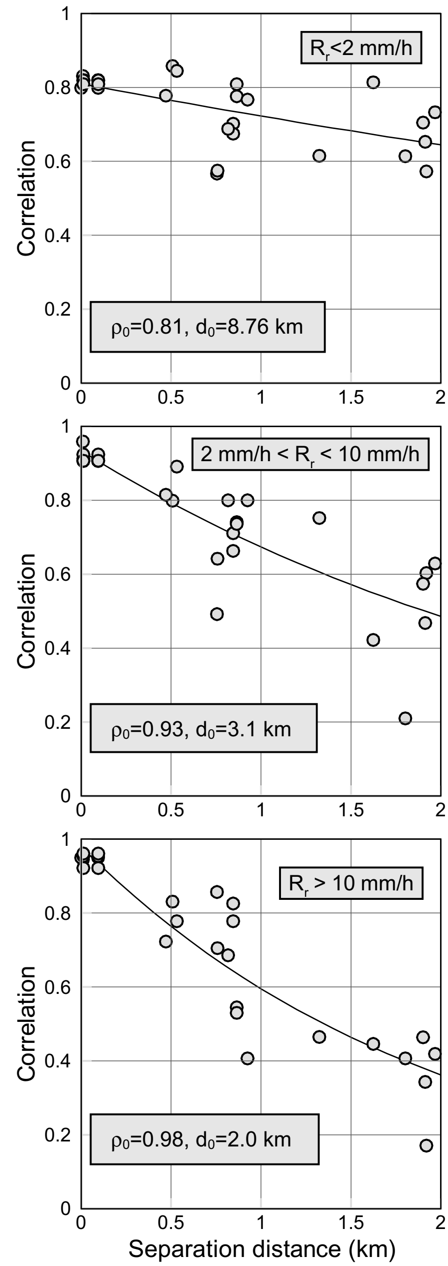

Fig. 8. The estimates of the conditional correlation functions based on the Florida sample. Conditioning is made on three ranges of radar rain-rate (R_r) values shown in the panels. The solid curves show the exponential fits to the inter-gauge correlation coefficients. Parameters of the exponential fitted function are also shown.

obtain some insight into the possible effects of sample size on the performance of the point-area transformation scheme, we performed a Monte Carlo (MC) experiment. We assumed that both point and areal rainfall follow the lognormal probability distribution. The relevance of the lognormal distribution is supported by several previous studies [20,29]. Assuming a certain mean and variance for the point rainfall distribution, we generated samples of point rainfall with sizes ranging from 50 to 1000 data points. The population mean and variance for these samples were 33 mm/h and 550 (mm/h)^2, respectively. The ratio of the variances of the areal and point rainfall (often called the "variance reduction factor") was chosen to be equal to 0.6. Thus, the corresponding areal rainfall has the population variance of 330 (mm/h)^2. These specific values correspond approximately to the parameters of one of the sub-samples that we analyzed in the previous section. We chose the sub-sample in the range of high rainfall intensities because strong rainfall is the most important from the practical point of view. On the other hand, the number of data points drops dramatically with increasing rain-rates and, thus, the sampling errors can be large in this range.

We applied the point-area transformation scheme described in Section 2 to the point rainfall samples in order to reproduce frequency distributions of the areal rainfall. A comparison of the transformed distributions versus the truth (the pre-assumed lognormal distributions of the areal rainfall) indicates the sensitivity of the transformation scheme to the sample size. Herein,

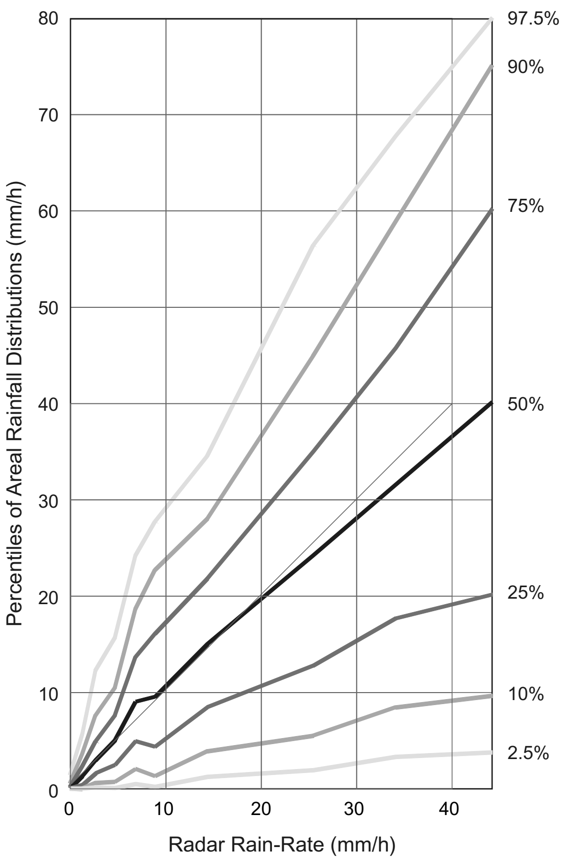

Fig. 9. Conditional distributions of the CDT-based areal rainfall as a function of the RR estimates for the Florida sample.

we compared the following parameters of the modeled (obtained from the point-area transformation) areal distribution against their corresponding true values: the mean, the variance, and the 2.5% and 97.5% percentiles. The results are summarized in Fig. 10 as the functions of sample size. The figure shows the ensemble averages and the 95% confidence intervals (CI) of the estimated parameters.

Fig. 10 also shows that the estimates of the mean and variance are consistent, as they should be. However, some biases for the distribution tails still remain, even for large sample sizes. We believe that this effect is a specifics of the CDT and that it does not have any physical meaning. Estimation of distribution tails is usually difficult due to the rapidly diminishing number of the data points, and this applies also to the CDT method. Note however that, from the practical point of view, the magnitude of these biases is rather small, especially in comparison with the distribution tails of the single-gauge rainfall that we presented in Section 5.

The simulation results indicate that, for sample sizes of a few hundreds, the parameters of the distributions of the areal rainfall retrieved using the transformation scheme are characterized by relatively high sampling errors, especially for the high order characteristics of the distribution (variance and 97.5% percentile). A sample size of 500–1000 is required for the transformation scheme to be able to reproduce the true areal distribution. Note that most of the results reported in Section 6 are based on sample sizes of about 100–500, as shown in Fig. 7. Thus, the uncertainty bounds shown in Fig. 9 are likely to be affected to some degree by the sampling errors. These effects are rather difficult to quantify accurately in practice. The simulation results discussed above can provide only rough information about the possible magnitude of the sampling problem because they are based on an idealized MC experiment with pre-assumed distributions for both the point and areal rainfall. In general, it seems that large sample sizes are required to obtain stable statistics of the areal rainfall distributions when conditioned on radar estimates. This is also necessary for the conditional estimation of the spatial correlation behavior, which is an important element in a successful implementation of the discussed CDT method. However, any reliable evaluation of RR products, including the estimation of the performance measures in the distribution-oriented verification methodology, also has to be based on large data samples. Thus, considering the major application of the CDT method, this requirement is an inherent part of the RR uncertainty problem.

8. Discussion

Successful application of the CDT method requires sufficiently accurate information on the rainfall spatial correlation structure conditioned on the radar estimates over the spatial scales below the resolution of the RR product. This information is available in many situations where dense raingauge clusters exist within the sparse operational networks. Estimating this correlation function directly based on the typical sparse raingauge networks is problematic. We believe that this problem can be solved by combining our increasing knowledge about spatial rainfall structure with an analysis of the RR fields in the verification data sample. One possible way to estimate the sub-scale correlation function could be through down-scaling of the spatial variability characteristics obtained from the verified RR products. More experimental research on small-scale rainfall variability will help us to work out satisfactory solutions to this practical implementation problem [22]. Of course, both the direct and indirect estimates of spatial correlation are bound to be uncertain [16]. However, the effects of these uncertainties on the CDT scheme are complex and their quantification requires an extensive investigation that is beyond this study. We will pursue these questions in our future research. However, one thing that we can be sure of is that, whatever are the uncertainties in the conditional

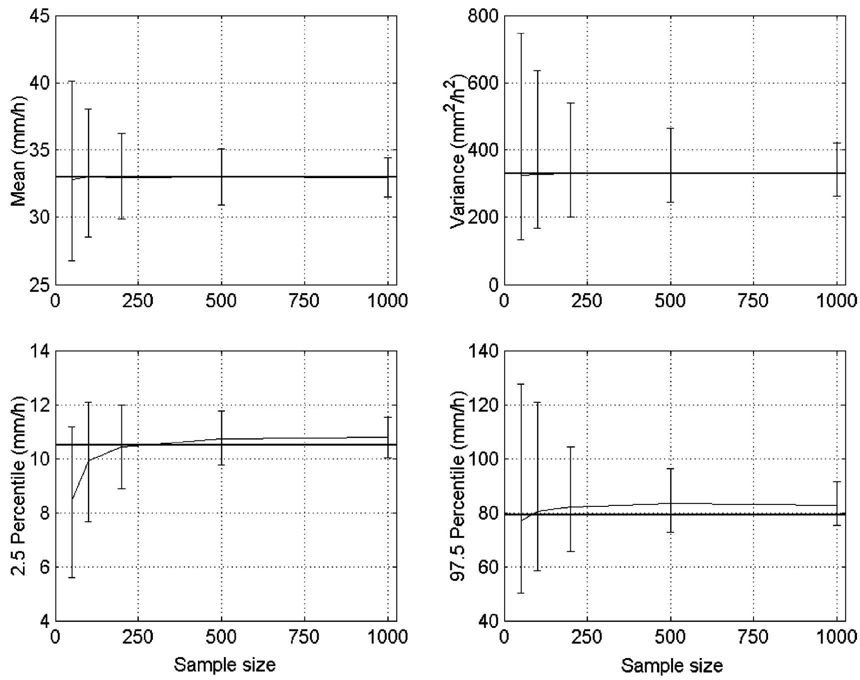

Fig. 10. Results of the Monte Carlo simulations demonstrating the sampling errors in the CDT method as a function of the sample size. The considered parameters are mean, variance, and 2.5 and 97.5 percentiles. The ensemble averages of the sample estimates of the transformed distribution parameters (dark solid curves) are compared against the assumed population parameter values (light horizontal lines). Vertical bars show the 95% CIs of the parameter estimates.

correlations, the CDT always reduces the RR uncertainty bounds in comparison with what we could obtain if we treated the single-gauge data as the corresponding truth. By its mathematical nature, the CDT just cannot increase the estimated RR uncertainty bounds. Of course, these RR uncertainty estimates are also not perfectly accurate, but this gauge-error filtering method always corrects them in the right direction. The effectiveness of this correction is clearly demonstrated in Fig. 5 by comparison of its results and the single-gauge performance. As one can see, the departures from the one-to-one line for the CDT transformed and the true areal rainfall distributions are about ten time smaller than the discrepancies between the single-gauge and true areal rainfall distributions. Error reduction by an order of magnitude is a very good performance for a relatively simple statistical method.

As we stated in Section 3, the method assumes that the spatial rainfall process is statistically homogenous over the analyzed area. We believe, that it does not have to be a perfect stochastic homogeneity. It is sufficient that the spatial differences in the first and second order statistics must be small enough so that the method still provides a significant improvement in comparison with the situation where single raingauges are treated as the "ground truth." And the performance tests in this study demonstrate clearly that the improvement is indeed considerable.

The conditional scaling factors in the CDT method and their dependence on the spatial averaging scale are determined only by the distributions and the spatial correlation functions of the radar-conditioned raingauge rainfall. These quantities are, in principle, measurable and no other fundamental assumptions are necessary to use the CDT in practical applications. This is in contrast to the classical multiscaling methods that are based on power-law relationships to transform the distributions from one scale to another [14,23,27, for example]. There is absolutely no evidence that there exist a spatial scaling law of this specific form that would be suitable for the range of scales from $100\,cm^2$ (raingauges) to $100\,km^2$ (small basins) that we have to deal with in our applications. All the published evidence of the "power-law scaling" in spatial rainfall is based on radar estimates that have the resolution of $1\,km^2$, at the best, and provide only some function of radar reflectivity, not real rainfall. And we know, from our experience, that the empirical investigation of this range of spatial scales is still problematic [22].

The Monte Carlo simulation experiment performed in this study suggests that the sample size requirements for our current implementation of the CDT method might be fairly high, especially in the cases of high rainfall intensities. This is not surprising since the extreme variability of rainfall and high level of the uncertainties make obtaining any stable estimates a challenging problem. We speculate that application of more advanced estimation methods [31] could improve considerably the estimation efficiency and reduce the sample size demands. This is important from the point of view of practical applications of the CDT method to the most common situations, in which the radar–gauge verification samples are of limited size. Implementation of more efficient statistical apparatus should also be a subject of future research.

9. Conclusions

We presented our first results on the development of a conditional transformation (CDT) method for estimation of the actual uncertainties in the inherently area-averaged RR products based on their comparisons with single raingauge data. We described the transformation scheme and tested its accuracy using large data samples from two high-density raingauge networks covering the Goodwin Creek watershed in Mississipi and the Little Washita watershed in Oklahoma. We also presented an example implementation of the CDT procedure in a quasi-operational situation using a two-month data sample from a NASA experiment in Florida. Finally, we carried out a Monte Carlo experiment to address the problem of sample requirements for the discussed point-area transformation method.

The tests performed in this study confirmed that the discussed point-area distribution transformation scheme was able to retrieve the areal rainfall distributions from the point measurements with satisfactory accuracy. This good performance was demonstrated both for the unconditional setup and for the full CDT scheme, in which the verification sample is stratified by different RR values. Although the full CDT procedure was tested only for the 15-min time-scale due to the limited sample size, we believe that our results validate the method also for longer accumulation intervals where the spatial rainfall variability is considerably smaller.

The purpose of the CDT method is to filter-out the raingauge representativeness errors from the typical radar–gauge verification samples. This raingauge error filtering allows analyzing statistical properties of any RR product in terms of the CDT-retrieved bivariate distributions of the radar estimates and the corresponding area-averaged rainfall. Such a representation enables successful application of the distribution-oriented verification methods that are already well established in the area of weather forecasting [19]. Additionally, it can provide a sound basis for building empirically supported models of RR uncertainties [11]. Thus, the CDT method, or any other scheme that can perform the gauge-error filtering with satisfactory accuracy, should be perceived as a prerequisite for more meaningful radar–raingauge comparisons.

Acknowledgments

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