3.5 Problems

(last update 14 June 2024)

1. Two family vehicles, a car and a truck, are classified by region of assembly. The three assembly regions are North America (N), Europe (E) and Asia (A). We will refer to vehicles assembled in North America as domestic and those assembled in Europe or Asia as foreign.

a. List the 9 elementary outcomes for this experiment (Explain your notation.).

b. List the outcomes in the event that one of the vehicles is domestic and the other is foreign.

c. List the outcomes in the event that at least one of the vehicles is foreign.

d. List the outcomes in the complement of the event in (c).

2. Each of four restaurants is classified as local (L) or in a national/regional chain (N).

a. List are the 16 elementary outcomes for this experiment (Explain your notation.).

b. List the outcomes in the event that exactly three of the restaurants are local.

c. List the outcomes in the event that all four restaurants are of the same type.

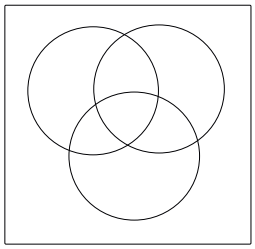
d. List the outcomes in the event that at most one of the four restaurants are local.

e. List the outcomes in the union of the events in (c) and (d), and list the outcomes in the intersection of these two events.

f. List the outcomes in the union of the events in (b) and (c), and list the outcomes in the intersection of these two events.

g. List the outcomes in the complement of the event in (d)?

A generic Venn diagram with three events.



3. A large construction company is currently working on three large projects (1,2,3). For $i = 1, 2, 3$, let A_i denote the event that project i is completed by its contract date. For each of the events described below, use the operations of unions, intersections, and complements to describe the event in terms of A_1, A_2, and A_3, draw a Venn diagram with three circles (similar to that above), and shade the region corresponding to the event.

a. At least one project is completed by its contract date.

b. All three projects are completed by their contract dates.

c. Only project 1 is completed by its contract date.

d. Exactly one project is completed by its contract date.

e. Project 1 is completed by its contract date or both of the other projects (but not all three) are completed by its contract date.

4. Suppose that a die is tossed twice. Express the following events as subsets of the set of 36 ordered pairs (see Table 3.1 reproduced below) in the sample space for this experiment. If possible describe the location of the ordered pairs instead of listing them, *e.g.*, the outcomes in column two correspond to observing a two on the second toss.

a. A "the number on the first toss is larger than the number on the second toss"

b. B "the number on the first toss is even"

c. C "the first toss yields a four"

d. D "the sum of the numbers on the two tosses is seven"

e. $A \cup C$

f. $A \cap D$

g. C^c

Table 3.1 Elementary outcomes (ordered pairs) when a die is tossed twice.

	second toss					
first toss	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

5. A vehicle arriving at an intersection can turn left, turn right, or go forward through the intersection. For the following questions let L denote "turns left", R denote "turns right", and F denote "goes forward through the intersection". Consider an experiment consisting of observing the movement of two vehicles through the intersection.

a. List the elementary outcomes in the sample space for this experiment.

b. List the outcomes in the event that both vehicles turn.

c. List the outcomes in the event that the vehicles turn in opposite directions.

d. List the outcomes in the event that the first vehicle turns left.

e. List the outcomes in the union of the events in (c) and (d), and list the outcomes in the intersection of these two events?

6. Suppose that one card is selected from a deck of 20 cards that contains 10 red cards numbered from 1 to 10 and 10 blue cards numbered from 1 to 10.

a. List 20 elementary outcomes in the sample space Ω of this experiment.

b. List the outcomes in the event A "a card with an even number is selected".

c. List the outcomes in the event B "a blue card is selected".

d. List the outcomes in the event C "a card with a number less than five is selected"

Describe each of the following events both in words and as a subset of Ω:

e. $A \cap B \cap C$

f. $A \cup B \cup C$

g. $A \cap (B \cup C)$

h. $B \cap C^c$

i. $A^c \cap B^c \cap C^c$.