4.4 Problems (last update 13 June 2024)

1. At a certain company, 60% of the employees are certified to operate machine A, 30% are certified to operate machine B, and 10% are certified to operate both machine A and machine B. Let A denote the event that a randomly chosen employee is certified to operate machine A, and let B denote the event that a randomly chosen employee is certified to operate machine B.

(a) Find the probability that a randomly chosen employee is certified to operate at least one of machines A and B.
(b) Find the probability that a randomly chosen employee is certified to operate one of machines A and B but not both.
(c) Find the probability that a randomly chosen employee is certified to operate machine A but not machines B.
(d) Find the probability that a randomly chosen employee is certified to operate at most one of machines A and B.

2. The water supplies of the cities in a state were tested for two kinds of impurities commonly found in water. It was found that 20% of the water supplies had neither sort of impurity, 40% had an impurity of type A, and 50% had an impurity of type B. If a city is chosen at random, what is the probability its water supply has exactly one type of impurity?

3. John is going to graduate from a university at the end of the semester. After being interviewed at two companies he likes, he assesses that his probability of getting an offer from company A is 0.8, and his probability of getting an offer from company B is 0.6. If he believes that the probability that he will get offers from both companies is 0.5, what is the probability that he will get an offer from at least one of these two companies?

4. An automobile manufacturer is required to recall all its cars manufactured in a given year for the repair of possible defects in the air–bag system and possible defects in the brake system. Dealers have been notified that 3% of the cars have defective air–bag systems only, and that 6% of the cars have defective brake systems only. If 87% of the cars have neither defect, what percentage of the cars have both defects?

5. From past experience, a stockbroker believes that under present economic conditions a customer will invest in tax–free bonds with a probability of 0.6, will invest in mutual funds with a probability of 0.3, and will invest in both tax–free bonds and mutual funds with a probability of 0.15. At this time, find the probability that a customer will invest
(a) in tax-free bonds but not mutual funds;
(b) in tax-free bonds or mutual funds (or both);
(c) in neither tax-free bonds nor mutual funds.

6. At a certain company, 40% of the employees are certified to operate machine A, 50% are certified to operate machine B, 40% are certified to operate machine C, 15% are certified to operate machines A and B, 10% are certified to operate machines A and C, 5% are certified to operate all three machines, and 15% are certified to operate machine C but neither machine A nor machine B. Let A denote the event that a randomly chosen employee is certified to operate machine A, let B denote the event that a randomly chosen employee is certified to operate machine B, and let C denote the event that a randomly chosen employee is certified to operate machine C.

For each of the following, include an expression for the event in terms of set operations on A, B and C, and justify your answer.

(a) Find the probability that a randomly chosen employee is certified to operate at exactly one of the three machines.
(b) Find the probability that a randomly chosen employee is certified to operate machine A and machine C but not machines B.
(c) Find the probability that a randomly chosen employee is certified to operate two of the three machines but not all three.
(d) Find the probability that a randomly chosen employee is certified to operate at least one of the three machines.

7. Among the students in a high school graduating class, 54% studied mathematics during their senior year, 69% studied history during their senior year, and 35% studied both mathematics and history during their senior year. If one of these students is selected at random, find the probability that during their senior year

(a) the student studied mathematics or history (or both);

(b) the student studied neither of these subjects;

(c) the student studied history but not mathematics.

8. Given events A and B, defined for the same sample space, is it possible to have $P(A) = \frac{1}{2}$, $P(A \cap B) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$?

9. Explain why $P(A \cup B) \leq P(A) + P(B)$.

10. Given events A and B, defined for the same sample space, with $P(A) = P(B)$ and $P(A^c \cap B^c) = P(A \cap B) = \frac{1}{6}$. Find

(a) $P(A)$.

(b) $P(A^c \cup B^c)$.

(c) the probability that exactly one of the events A or B occurs.