7.10 Problems

(last update 14 June 2024)

1. Let X denote a discrete random variable with probability mass function (p.m.f.)

$$p(x) = \begin{cases} 2c & \text{for } x = 1 \\ 4c & \text{for } x = 2 \\ 6c & \text{for } x = 3 \\ 8c & \text{for } x = 4 \\ 7c & \text{for } x = 5 \\ 5c & \text{for } x = 6 \\ 3c & \text{for } x = 7 \\ 1c & \text{for } x = 8 \\ 0 & \text{otherwise.} \end{cases}$$

where c is a suitable constant. **Do not round off – express all values as rational fractions.**

a) Find the value of c for which this is a valid probability mass function.

b) Find the probability $\mathrm{P}(X \leq 5)$.

c) Find the probability $\mathrm{P}(2 < X \leq 5)$.

d) Find the expected value of X.

e) Find the expected value of X^2.

f) Find the variance of X.

2. Consider a box containing 3 red balls and 7 white balls. Suppose that balls are drawn one at a time, at random, without replacement from this box until two red balls are obtained. Let X denote the number of the draw on which the second red ball is obtained.

a) Find the probability mass function (p.m.f.) of X.

b) Find the expected value of X.

c) Find the expected value of X^2.

d) Find the variance of X.

3. Assume that the probability that a letter will be delivered within three working days is .9. Suppose that 10 letters inviting friends for a dinner party are sent out on Tuesday. Everyone who receives the invitation by Friday (*i.e.*, within 3 working days) will come. Those who do not receive the invitation by Friday will not come. Let X denote the number of friends who come to dinner.

(a) What is the name of the distribution of X?
(b) Find the probability that at least 7 friends will come.
(c) What are the expected value and variance of X.
(d) If the caterer charges a base fee of \$100 plus \$10 for each guest who comes to the dinner party. What is the expected value and variance of the catering cost?

4. A jewelry dealer is considering the sale of an antique necklace. She estimates that there is a 20% chance that she will make a profit of \$250 on the sale, a 40% chance that she will make a profit of \$150 on the sale, a 30% chance that she will break even on the sale, and a 10% chance that she will lose \$50 on the sale. What is her expected profit?

5. A manufacturer ships parts in lots of 1000 and makes a profit of \$50 per lot sold. The purchaser, however, subjects the product to a sampling plan as follows: 10 parts are selected at random with replacement. If none of these parts are defective, the lot is purchased; if one part is defective, the lot is purchased but the manufacturer returns \$10 to the buyer; if two or more parts are found to be defective, the entire lot is returned at a net loss of \$25 to the manufacturer. What is the manufacturers expected profit if 10% of the parts are defective?

6. A jury trial ended in a hung jury with 8 jurors voting guilty and 4 jurors voting for acquittal. Suppose that the 12 jurors exit the courtroom in random order. How many of the first four jurors to leave the courtroom would be expected to have voted for acquittal? In other words, what is the expected value of X, when X denotes the number of jurors out of the first four jurors to leave the courtroom who voted for acquittal?

7. A salesman has scheduled two sales appointments. The probability that he will be able to close the deal at his first appointment is .75. The probability that he will be able to close the deal at his second appointment is .45. If he closes the deal at the first appointment he will earn a commission of \$1,000 on that sale and, regardless of the outcome of the first appointment, if he closes the deal at the second appointment he will earn a commission of \$1,500 on that sale. Assuming that the outcomes of the deals at the two appointments are independent, what is the salesman's expected profit?