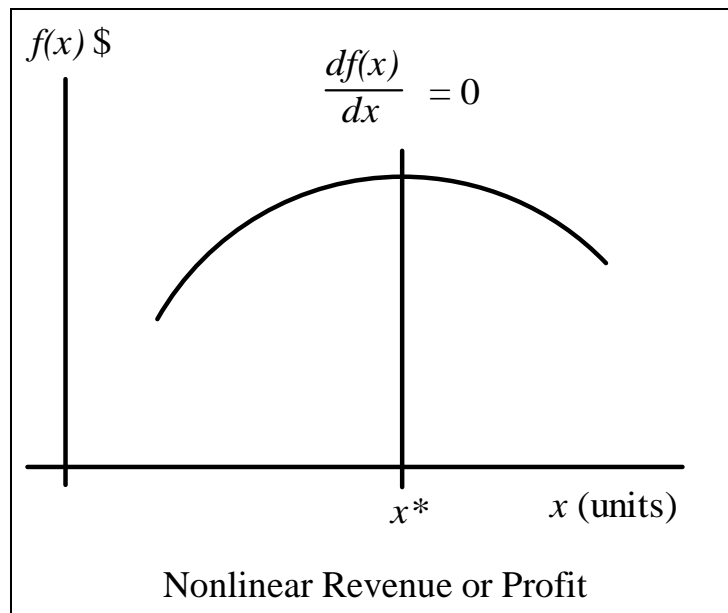
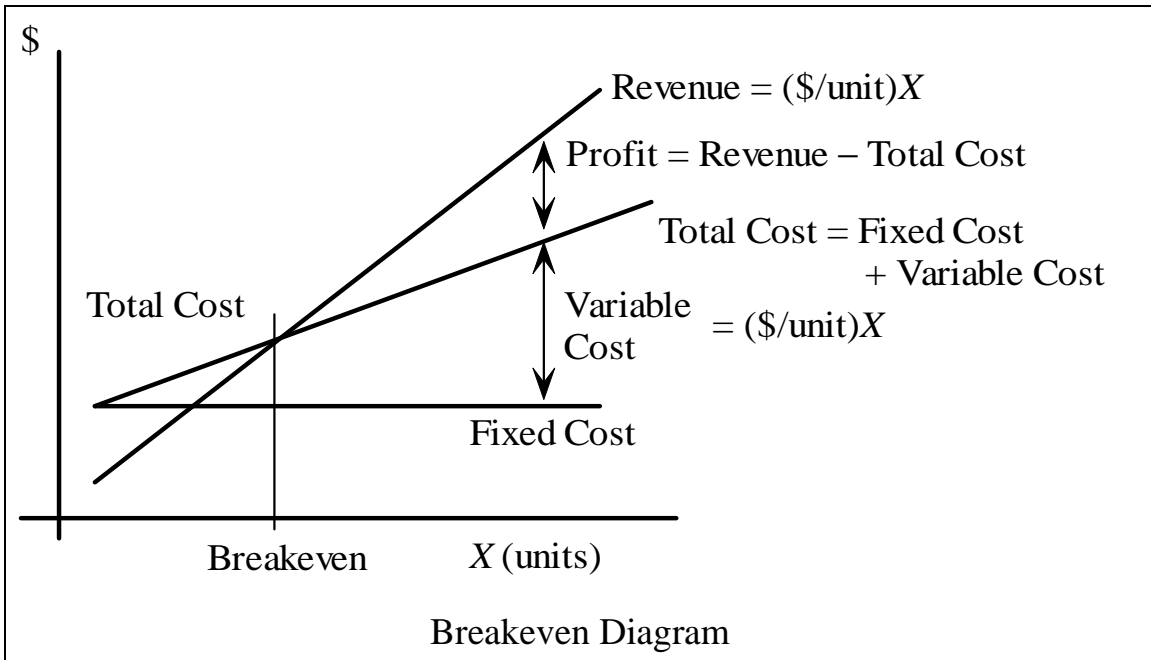
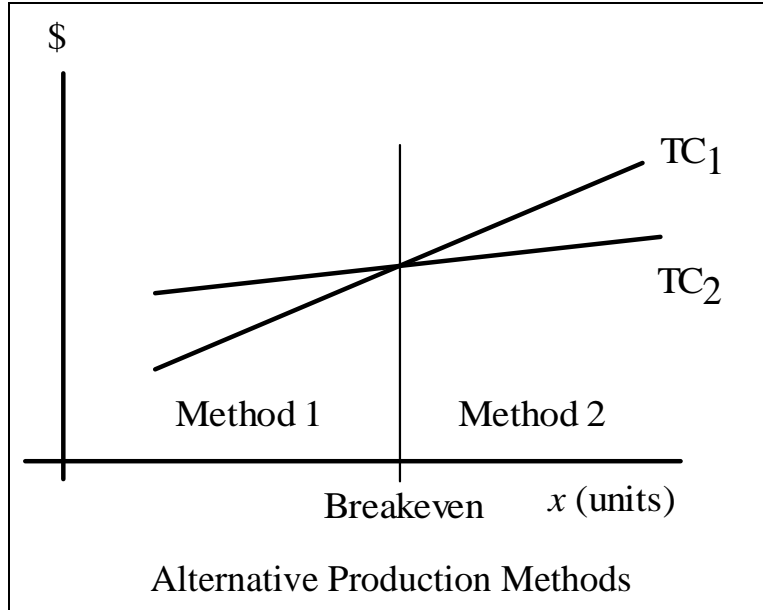


# FE Review for Engineering Economics

<http://ristroph.louisiana.edu/EngrEcon/Overhead/FEReview.pdf>

## Present Economics





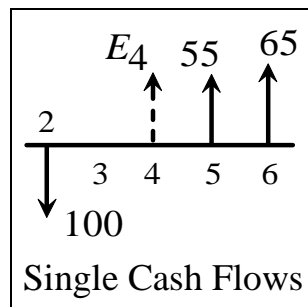
Expected Profit or Cost

$$P(100) = 30\% \quad P(200) = 20\% \quad P(300) = 50\%$$

$$\text{Expected value} = 220 = 100(0.30) + 200(0.20) + 300(0.50)$$

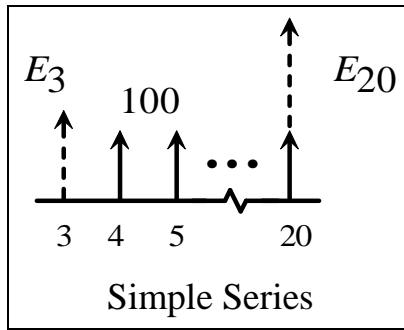
**Equivalence**

Single Payments



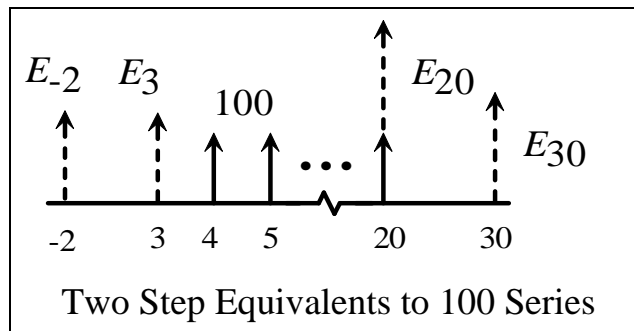
$$E_4 = -100(F|P, i, 4-2) + 55(P|F, i, 5-4) + 65(P|F, i, 6-4)$$

Uniform Series



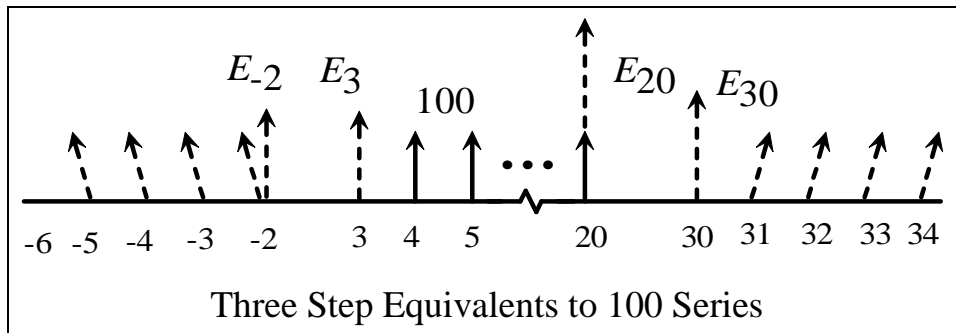
$$E_3 = 100(P|A, i, 20-3)$$

$$E_{20} = 100(F|A, i, 20-3)$$



$$E_{-2} = E_3(P|F, i, 3-(-2))$$

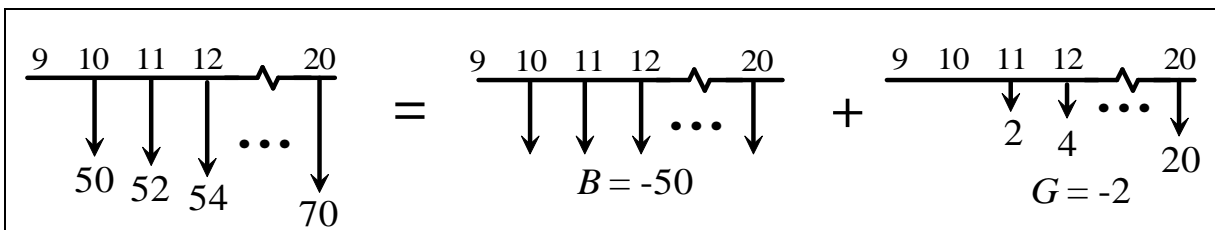
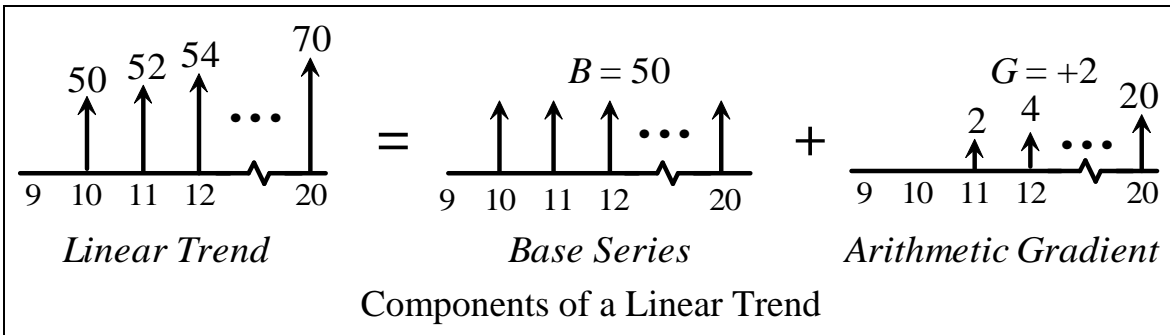
$$E_{30} = E_{20}(F|P, i, 30-20)$$



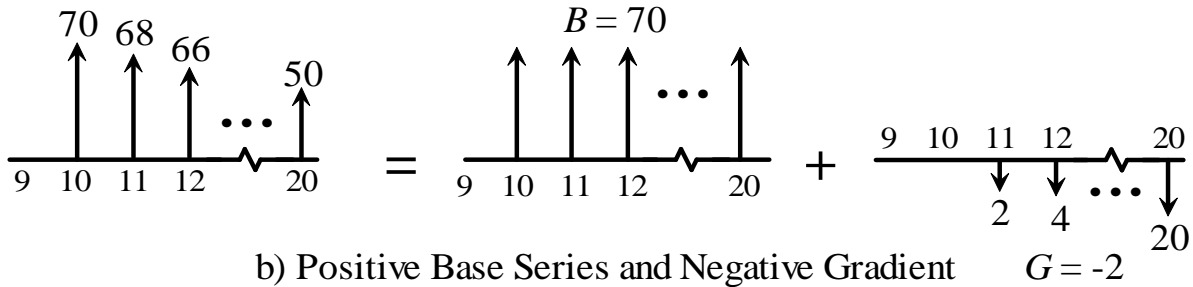
$$E_{-5 \text{ to } -2} = E_{-2}(A|F, i, -2-(-6))$$

$$E_{31 \text{ to } 33} = E_{30}(A|P, i, 34-30)$$

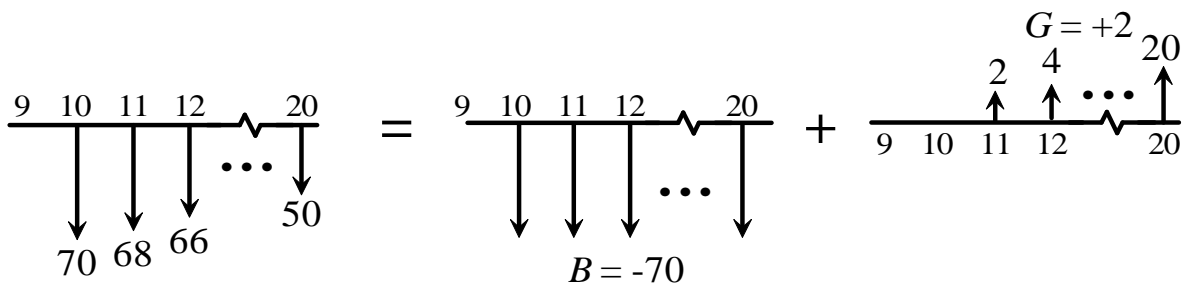
Arithmetic Gradients



a) Negative Base Series and Negative Gradient

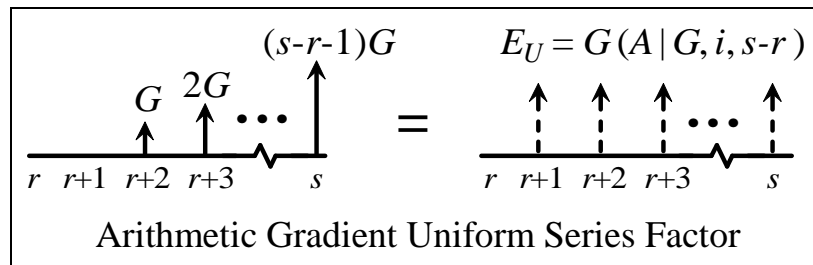
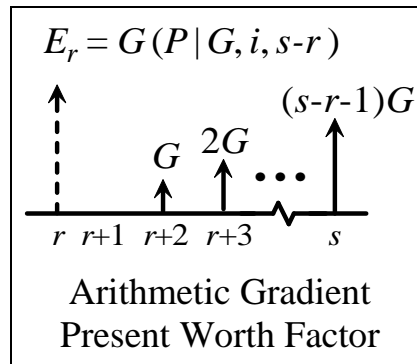


b) Positive Base Series and Negative Gradient



c) Negative Base Series and Positive Gradient

Types of Linear Trends



### Effective Interest

$r$  = nominal rate, such as 12% per year compounded monthly

$P$  = number of short periods in the long period, such as 12 months in a year

$i_P = r/P$  = rate per short period, such as a month

$$i_e = (1 + i_P)^P - 1 = \text{effective (or true) interest rate for the long period, such as a year}$$

Example: 18% per year compounded monthly

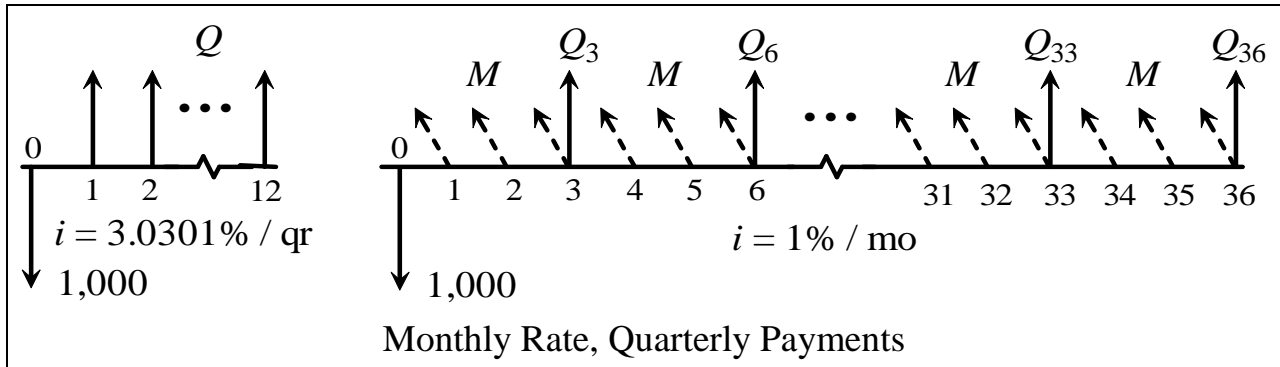
$$P = 12$$

$$i_m = 1.5\% \text{ per month} = 18\% / 12 = \text{true monthly rate}$$

$$i_e = (1 + 0.015)^{12} - 1 = 19.56\% \text{ effective (or true) interest rate per year}$$

Example: 12% per year compounded monthly with quarterly cash flows

=> The monthly interest rate is 1%



❖ If draw cash flow diagram on a quarterly basis, must use quarterly rate

$$3.0301\% = (1 + 0.01)^3 - 1 \quad \leq 3 \text{ months per quarter}$$

$$Q = 1000(A|P, 3.0301\%, 12-0)$$

❖ If draw cash flow diagram on a monthly basis, get monthly payments

$$E_m = 1000(A|P, 1\%, 36), \quad \text{Step 1}$$

Since lender only getting quarterly payments, then must pay interest on missed monthly payments

$$Q_3 = E_m(F|A, 1\%, 3-0), \quad Q_6 = E_m(F|A, 1\%, 6-3)$$

$$\dots \quad Q_{36} = E_m(F|A, 1\%, 36-33)$$

In general,

$$Q = E_m(F|A, 1\%, 3-0) \quad \text{Step 2}$$

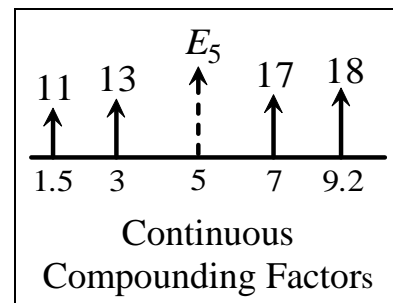
❖ Continuous Compounding

$$(P | F, r, n) = e^{-r n}$$

$$(F | P, r, n) = e^{r n}$$

$$\$58.58 = 11 e^{0.1(5-1.5)} + 13 e^{0.1(5-3)}$$

$$+ 17 e^{-0.1(7-5)} + 18 e^{-0.1(9.2-5)}$$



## Present Worth

- ❖ All cost and benefits given: draw cash flow diagrams and compute PW. Equal lives not necessary.
- ❖ Only costs given: equal benefits or known difference in benefits must be assumed.
  - Planning horizon can be truncated. For example, 3 and 4 year pumps can be compared over a 7 year planning horizon. Just draw the cash flows for the 7 years, compute the PC's, and choose the smallest.
  - Planning horizon can be a common multiple with fully repeating, identical life cycles. For example, 3 and 4 year pumps can be compared over a 12 year planning horizon. Compute the EAC of the 3 year pump over its 3 year life cycle, and the EAC of the 4 year pump over its 4 year life cycle. Then for each pump

$$EAC_{\text{Over 12 years}} = EAC_{\text{Life Cycle}}$$

$$PC_{\text{Over 12 years}} = EAC_{\text{Over 12 years}}(P|A, i, 12-0)$$

- "Capitalized cost" means present cost, usually of an infinite series.

$$(P|A, i, \infty) = 1/i$$

Equivalent is one period before the first series flow

## Equivalent Annual Worth

- ❖ All cost and benefits given: draw cash flow diagrams and compute EAW. Equal lives not necessary, but compute all EAW's over same time period, usually the life of the longest alternative
- ❖ Only costs given: equal benefits or known difference in benefits must be assumed.
  - Planning horizon can be truncated. For example, 3 and 4 year pumps can be compared over a 7 year planning horizon. Just draw the cash flows for the 7 years, compute the EAC's, and choose the smallest.
  - Planning horizon can be a common multiple with fully repeating, identical life cycles. For example, 3 and 4 year pumps can be compared over a 12 year planning horizon. Compute the EAC of the 3 year pump over its 3 year life cycle, and the EAC of the 4 year pump over its 4 year life cycle. Then for either pump

$$EAC_{\text{Over 12 years}} = EAC_{\text{Life Cycle}}$$

- If the planning horizon is infinite

$$(A|P, i, \infty) = i$$

Equivalent series begins one period after original cash flow

- ❖ Heuristic: long and *indefinite* planning horizon with repeating life cycles:

$$EAC_{\text{Planning Horizon}} \approx EAC_{\text{Life Cycle}}$$

### Internal Rate of Return

- ❖ Solve  $PW = 0$ . Reject if  $IRR < AMRR$  (MARR)
- ❖ If any alternatives left:
  - Rank in order of increasing first cost
  - On a pair-wise basis, compute Higher Cost – Lower Cost cash flows
  - Solve  $PW_{\text{High} - \text{Low}} = 0$  to compute NIRR
    - Choose High if  $NIRR \geq AMRR$  (MARR)
    - Choose Low if  $NIRR < AMRR$  (MARR)
- ❖ Comparing pure cost ventures, such as pumps, using NIRR
  - Cannot compute IRR's, so skip that step
  - Rank and do pairwise comparisons as above

### Benefit Cost

- ❖ Compute  $PB / PC$ . Reject if  $PB / PC < 1$
- ❖ If any alternatives left:
  - Rank in order of increasing *present* cost
  - On a pair-wise basis, compute

$$NBC = \frac{PB_{\text{High}} - PB_{\text{Low}}}{PC_{\text{High}} - PC_{\text{Low}}}$$

- Choose High if  $NBC \geq 1$
- Choose Low if  $NBC < 1$

## Special Topics for PW, EAW, or IRR

### ❖ Bonds

- Redemption period or bond life is when its last payment is made
- Face, redemption, or par value is what owner receives at end of bond life
- Coupon rate or bond's rate is used to compute the periodic payment or coupon

$$\text{Coupon} = \text{Face Value} \times \text{Coupon Rate}$$

- Maximum to pay is present worth computed at buyer's discount rate
- Example: 10 year bond with 1,000 face value and 8% coupon rate. Buyer can earn 12% elsewhere

$$\text{Max} = 1,000(0.08)(P|A, 12\%, 10-0) + 1,000(P|F, 12\%, 10-0)$$

- Example: Similar bond, but third payment just made

$$\text{Max} = 1,000(0.08)(P|A, 12\%, 10-3) + 1,000(P|F, 12\%, 10-3)$$

- Example: "Zero coupon," 10 year \$1,000 bond that makes no payments and current time is 3

$$\text{Max} = 1,000(P|F, 12\%, 10-3)$$

### ❖ Stocks

- Estimate cash flows from dividends and sale of stock
- Max to pay = PW of those cash flows

### ❖ Sunk Costs

- Always ignore costs that have already occurred. Decisions can be made only regarding choice of present and future cash flows

### ❖ Loan Payoffs

- Suppose 10 year loan with yearly payments at 8%:

$$\text{Note} = (\text{Amount Borrowed})(A|P, 8\%, 10)$$

- Suppose at time 3 and *just* made payment. Then discount remaining payments

$$\text{Amount Owed} = \text{Note}(P|A, 8\%, 10-3)$$

## Replacement

### ❖ Cash Flows

- Defender: Initial cost is its current market value, not its book value or trade-in value. Yearly costs are operations and maintenance costs, and then salvage at end of life is usually a benefit but can be a cost
- Challengers: Initial cost of each challenger is its installed cost. It also has yearly O&M costs, and salvage at end of life

### ❖ Fixed planning horizon with at most one replacement

- Draw cash flow diagrams over planning horizon, compute either PC or EAC and choose best alternative

### ❖ Multiple replacements

- # Possibilities =  $(1 + \# \text{ Challengers})^n$
- Try to find best possibilities using EAC and common sense

### ❖ Heuristic for keeping defender at least one more year or replacing today

- Assumes identical replacements with long, *indefinite* horizon
- For each alternative, compute its *economic life*. Use trial and error to compute the EAC's for several years. Economic life occurs where the EAC is the smallest. Call this value EAC\*.
- Compare every EAC\* and choose the smallest one. If it is a challenger, replace the defender today with that challenger. If it is the defender, keep it for at least one more year with exact timing determined using defender's marginal cost

## Depreciation

$BV_0 = \text{Installed Cost}$

$$BV_j = BV_0 - D_1 - D_2 \dots - D_j$$

$$BV_j = BV_{j-1} - D_j$$

$n = \text{recovery period or depreciation life}$

$SV = \text{legal salvage value}$

## ❖ Straight line

$$D_j = (BV_0 - SV) / n$$

## ❖ Declining Balance

- Declining balance rate = 100%, 125%, ..., 200% (double dec. bal.)
- $p = \text{DBR} / n$
- $D_j = pBV_{j-1}$  until reach  $SV$

❖ Optimal switch from DB to SL for  $SV = \$0$ : first  $D_j$  using SL when

$$j \geq n / \text{DBR} + 1$$

- Switch at time  $j-1$  so SL is used during year  $j$  and first SL charge at time  $j$

## ❖ MACRS

- Look up recovery rate  $r_j$  in a table
- $D_j = r_j BV_0$
- Early retirement: mid-year for personal and mid-month for real

## Taxes

## ❖ ATCF = BTCF – Cash Flow for Taxes

❖  $t$  = tax rate (depends on bracket)

- Always use rate for current bracket, never average rate
- Applied to taxable income

❖ Ordinary income taxes = Bracket Amount +  $t$  (Amount in Bracket)

- Business: Taxable Income = Revenue – Expenses  $\pm$  GLD
  - Cannot deduct a purchase, only depreciation
- Personal: Taxable Income = Gross Income (includes GLDs) –  
Adjustments – Personal Exemptions – Deductions
  - Deductions = max(Itemized deductions, Standard deduction)

- ❖ Gain or loss on disposal laws are complex. If \$0, then ignore.
- ❖ Effective Tax Rate
  - = Federal Rate + State Rate – (Federal)(State), state deductible from fed
  - = (Fed + State – 2×Fed×State) / (1 – Fed×State), if mutually deductible

## Inflation

- ❖ Nominal dollars in year  $j$  are the *number* of dollars:  $N_j$
- ❖ Real dollars in year  $j$  refers their purchasing power relative to some earlier year:  $R_j$ 
  - Suppose that \$100 dollar bills are required to purchase goods in 2010 that can be purchased for \$60 in 2000. Then the \$100 nominal dollars handed to the vendors in 2010 have a value of \$60 real dollars relative to 2000.
- ❖ Inflation rate is the percentage increase in prices each year:  $f$

$$N_j = (1+f)^j R_j$$

- If inflation averages 5.24% from 2000 to 2010, how much will goods costing \$60 in 2000 cost in 2010? Call year 2000 time 0, then

$$100 = (1+0.0524)^{10} 60$$

- ❖ Real dollars can be computed using  $R_j = N_j / (1+f)^j$ 
  - If inflation averages 5.24% from 2000 to 2010, how many real dollars worth of goods relative to 2000 will \$100 nominal dollars in 2010 purchase?

$$60 = 100 / (1+0.0524)^{10}$$

- ❖ Economic measures
  - Nominal or inflation adjusted discount rate is  $n = f + r + rf$ , where  $r$  is the real discount rate (AMRR based on real dollars)
  - Real discount rate is  $r = (n - f) / (1 - f)$ , where  $n$  is the nominal discount rate (AMRR based on nominal dollars)

- $PW_R = PW_N$
- Other measures: Use nominal rate on nominal dollars and real rate on real dollars.
- ❖ Multiple rates: use rates for each sector to compute net nominal and then deflate to real using general (CPI or PPI) deflator
- ❖ After tax analyses with inflation
  - Estimate all BTCF cash flows as nominal dollars
    - Remember that depreciation charges are fixed by law, so they do not change
    - Gains or losses on disposal must be computed using nominal dollars
  - Compute the ATCF as nominal dollars
  - Deflate to real dollars (or directly discount the ATCF using the inflation adjusted discount rate)

Good luck! Let me know how you do when you take the FE Exam. Drop me an email and let me know the types of questions being asked so that this review can be kept current. ☺ [ristroph@louisiana.edu](mailto:ristroph@louisiana.edu)