1. Show that $C^*_r(F_2)$ has strict comparison. Equivalently, $Cu(C^*_r(F_2))$ is almost unperforated (which in turn happens if and only if $Cu(C^*_r(F_2)) = \mathbb{Z}$).

2. Given a Cu-semimodule $M$ over a Cu-semiring $S$, is there a theory of crossed products?

3. Let $A$ be a counterexample to Naimark’s problem. What is $Cu(A)$?

4. Let $S$ be a Cu-semigroup, and let $x \in S$. Is it true that $x = \sum_{i=1}^{\infty} x_i$, where $x_i \ll \tilde{x}_i$ for some $\tilde{x}_i$ and all $i$? (This is related to the Corona Factorization Property. The answer was shown to be affirmative if $S$ is algebraic and has (O5), by Joan Bosa and Henning Petzka.)

5. Can an enriched Cuntz semigroup that contains $K^\text{alg}_1$ and Cu be used to classify all one-dimensional NCCW complexes and their inductive limits?

   (i) In stable rank one algebras, classify unitaries up to approximate unitary equivalence.

   (ii) Given $\varphi, \psi: C(T) \to A$, what does it mean that $Cu(\varphi) = Cu(\psi)$? (Again, if $A$ has stable rank one.)

   (iii) Classify injective maps $\varphi: C([0,1]^2) \to A$, for $A$ with stable rank one.

6. Study inf-semilattice ordered Cu-semigroups, i.e. Cu-semigroups with infima that satisfy $(x \wedge y) + z = (x \wedge z) + (y \wedge z)$. What is the Cuntz semigroup of Villadsen algebras of the first kind?

7. (i) Is $V(C(S^2)) \sqcup (0, \infty] = Cu(A)$ for some $A$?

   (ii) Let $Z' = \{0, 1_a, 1_b, 2, 3, \ldots \} \sqcup (0, \infty]$, where $1_a + 1_b = 2$. Is $Z' = Cu(A)$ for some $A$?

8. Are there examples for Villadsen algebras (of the first or second kind) having at least three of the following properties:

   (i) have a unique trace

   (ii) have stable rank one

   (iii) have finite radius of comparison

   (iv) have real rank zero?

9. Define Cuntz semigroup-like objects for

   (i) Non-selfadjoint operator algebras
(ii) Real C*-algebras (Luis: This exists already)
(iii) $L_p$ operator algebras
(iv) General rings.

10. For a Cu-semigroup $S$, define $\text{rank}(S) = \min\{k \mid S^\otimes k$ is $Z$-stable\}, and $\infty$ if no such integer exists. Study its properties.

11. What is the range of $A \mapsto \text{Cu}(A)$ for AI C*-algebras?

12. Let $A$ and $B$ be nuclear and separable such that $T(A) \cong T(B)$ (cones of $[0, \infty]$-valued l.s.c. traces). Does it follow that $A \otimes W \otimes K \cong B \otimes W \otimes K$?

13. Can one characterize stable rank one in terms of the Cuntz semigroup? Are there C*-algebras $A$ and $B$ such that $\text{Cu}(A) \cong \text{Cu}(B)$ with $A$ of stable rank one, but not $B$?

14. When does $\text{Cu}(A)$ have suprema of finitely many elements?